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APPROXIMATE DEPENDENCE OF THE DYNAMIC ANGLE OF REPOSE ON THE STATIC ANGLE OF REPOSE OF BULK MATERIAL

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Abstract: *The article state the relationship between static angle of repose and dynamic angle of repose of bulk materials. The relationship is based on published relationship between angle of repose (angle of slope of bulk material) and angle of internal friction. This published relationship was modified using measured static angles of repose and dynamic angle of repose. Relationship is valid only for angles above ~30°. From given data also ensue: If the static angle of repose approaches 30° the difference between static and dynamic angle of repose reduces. Similarly the angle of repose is equal to the angle of internal friction in the case ideal bulk material. The angle of repose and the angle of internal friction of ideal material is equal to 30°.*

Key words: *Angle of repose, Static angle of repose, Dynamic angle of repose, Angle of internal friction, Ideal bulk material*

1 INTRODUCTION

The purpose of the article is finding the relationship between the static angle of repose and the dynamic angle of repose of bulk materials. The aim is to determine the dynamic angle of repose only on the basis of measuring the static angle of repose.

2 BASIS FOR THE DEDUCTION RELATIONSHIP

In the textbooks [2] is stated empirical relationship between angle of repose and angle of internal friction. According to this relationship it is possible to calculate angle of internal:

$$tg(\varphi) = 0.9\ tg(\alpha), \qquad (1)$$

where φ is angle of internal friction and α is angle of slope of bulk material. Number 0.9 is value of empirical coefficient.

This relationship (1) was similarly used in case of dynamic angle of repose and static angle of repose:

$$tg(\varphi') = K\ tg(\alpha'), \qquad (2)$$

where φ' is dynamic angle of repose, α' is static angle of repose bulk material and K is empirical coefficient.

Static angle of repose and dynamic angle of repose of various bulk materials are given in the following ***Tab. 1***. Angles were measured according to publication [1]. Static angle of repose measurement lies in the angle of slope of bulk material measurement in a slowly rotating pipe at the moment of transition from standstill into movement – just before bulk material slides. Dynamic angle of repose measurement consists in angle of slope of bulk material measurement in a slowly rotating pipe at the moment of transition from movement into standstill – just after bulk material slides.

Tab. 1 *Static angles of repose and the dynamic angles of repose*

Bulk material	Static angle of repose α'/°	Dynamic angle of repose φ'/°
Coarsely granular inorganic salt[1)]	40.5	34.8
Granular inorganic salt[1)]	37.8	34.0
Finely granular inorganic salt[1)]	36.4	33.9
Very finely granular inorganic salt[1]	54.0	42.3
Finely granular coffee[1)]	45.0	38.3
Granular polyethylene[2)]	39.0	35.0
Scales paraffin[2)]	60.0	45.0

[1)] - own measurement

[2)] - measured values come out of the publication [1]

Empirical coefficients K were calculated of measured angles in ***Tab. 1***:

$K = tg(\varphi') / tg(\alpha')$ (3)

The calculated coefficients *K* are presented in ***Tab. 2***.

Tab. 2 *Coefficients K*

Bulk material	Empirical coefficient *K*/1
Coarsely granular inorganic salt	0.817
Granular inorganic salt	0.871
Finely granular inorganic sal	0.912
Very finely granular inorganic salt	0.660
Finely granular coffee	0.788
Granular polyethylene	0.864
Scales paraffin	0.577

The following *Fig.1* shows the dependence of the empirical coefficient on static angle of repose.

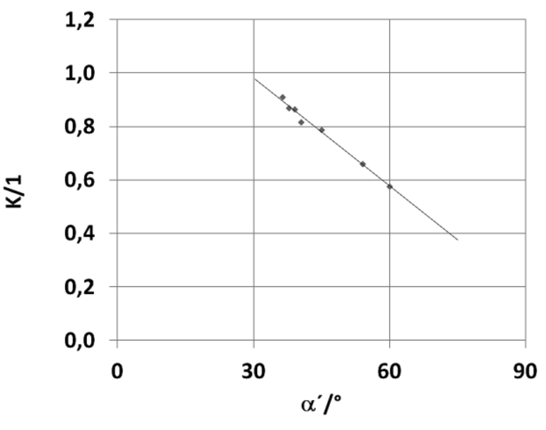

Fig.1 *Dependence of the empirical coefficient on static angle of repose*

Expression of the dependence of the empirical coefficient on static angle of repose:

$K = - 0.0135 \cdot \alpha' + 1.385$ (4)

3 RESULTS AND DISCUSSION

Angle of repose (angle of slope of bulk material) and simultaneously angle of internal friction of ideal bulk material is equivalent to 30° according to the article [3]. The more the bulk material approaches solid material the more the angle of internal friction rises. Angle of internal friction of solid material approaches 90°. The more bulk material approaches liquid material the more angle of internal friction goes down. Angle of internal friction liquid material approaches 0°.

If static angle of repose is 30° than according to the dependence in *Fig.1* and relationship (4) empirical coefficient is 0.989 and dynamic angle of repose is 29.5° according to relationship (3).

Expression of the dependence of dynamic angle on angle of repose is based on the relationship (1) and (3):

$$\varphi' = arctg((1.385 - 0.0135 \cdot \alpha') \cdot tg(\alpha')) \qquad (5)$$

The following *Fig.2* shows measured dynamic angles of repose and dynamic angles of repose calculated according to relationship (5) comparison.

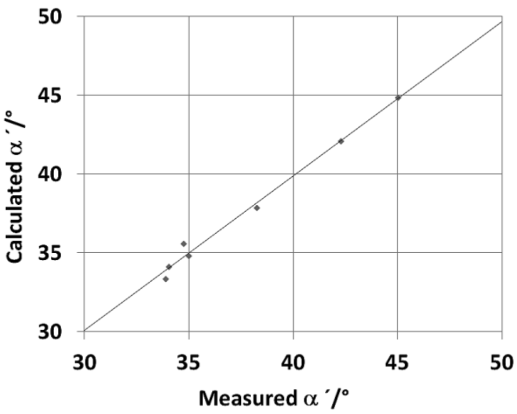

Fig.2 *Measured dynamic angles of repose and calculated dynamic angles of repose comparison*

4 CONCLUSIONS

Dependence of dynamic angle of repose on static angle of repose (5) was obtained on the basis of the measured data. Dependence is valid only for angles above ~ 30°. For angles less than ~ 30° may apply other dependence.

If the static angle of repose is 30° than empirical coefficient *K* approaches 1 and the dynamic angle of repose is approximately the same size as static angle of repose. This is consistent with the statement of article [3] despite the fact that angle in article [3] were measured other methods. Angle of repose (angle of slope of bulk material) is usually measured as angle between the pour bulk material slope and the base. Angle of internal friction is usually measured on the sparing machine.

References

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