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LOCATION CHOICE OF FIRMS UNDER STACKELBERG INFORMATION ASYMMETRY

***Serhij Melnikov* [1,2]**

[1] *Institute of Maritime Business, Odessa National Maritime University, Ukraine, tel: +380934437389, e-mail: nfn333@ukr.net*

[2] *Department of Transport Services Market, Institute for Market Problems and Economic-and-Ecological Research, Ukraine, tel: +380934437389, e-mail: nfn333@ukr.net*

Abstract:
This paper develops the spatial duopoly model [Liang, W.J., Hwang, H. and Mai, C.C., 2006, "Spatial discrimination: Bertrand vs. Cournot with asymmetric demands," Regional Science and Urban Economics, 36, pp. 790–802] to analyze the location choice of firms under Stackelberg information asymmetry. The competitive game consists of two stages. In the first stage, the firms simultaneously select their locations. In the second stage, at the given location decisions, the firms simultaneously choose their supplied quantities. The equilibrium of the model is solved by backward induction. It is obtained that under certain conditions the information asymmetry effect dominates the market size effect and both firms agglomerate in the small market. In this case the optimal location choice of firms depends on who makes the first move in the game.
Key words:
spatial duopoly, asymmetry, agglomeration

INTRODUCTION

After the emergence of the famous Hotelling's work [1], the problems of agglomeration and dispersion of firms in space have become a constant subject of economists' study. In case of price competition firms will disperse, as under agglomeration their profits will decrease until they become zero due to the Bertrand paradox [2]. In case of quantitative competition, firms will tend to agglomerate [3], [4].

The paper [5] studies the effects of spatial price discrimination on output, welfare and location of a monopolist in the context of spatial economy. It is shown that a monopolist will be located in different markets under different pricing schemes. In particular, if the slope of the demand function in one market is higher than in the other one, the monopolist will be located in the first market under simple pricing, or in the second market under discriminatory pricing.

Investigation of agglomeration and dispersion of firms depending on transport costs and market sizes is carried out in the paper [6]. This paper [6] develops a barbell model [5] with homogeneous product and asymmetric demands to compare prices, aggregate profits and social welfare between Cournot and Bertrand competition, and to analyze the firms' equilibrium locations. It focuses on the impacts of the spatial barrier generated from transport costs, and the market size effect resulting from asymmetric demands. It shows that the market-size effect is crucial in determining firms' locations under Cournot competition, but insignificant under Bertrand competition.

The paper [7] considers a spatial discrimination Cournot model with asymmetric demand. The model uses a geographic interpretation of the linear market and deals with differentiated products. The paper analyzes the quantitative and spatial solutions of firms and shows that agglomeration or dispersed location may occur depending on the combination of parameters.

As it is known, the effects of asymmetry are promising areas for studying spatial models [8]-[10]. The aim of this article is to develop the spatial duopoly model [6] and to analyze the location choice of firms under Stackelberg information asymmetry.

1 THE MODEL

Suppose there are two markets, which are located at the endpoints of the line with a unit length. The markets are connected by road or highway.

There is a size asymmetry between markets. Assume that the size of the left market (L-market) exceeds the size of the right market (S-market). There are two competing firms, which can be located at any point along a line. In both markets, firms sell homogeneous goods and arbitrage among consumers is excluded. Each firm faces linear transportation costs of t to move one good unit per one unit of distance.

A distance of the i^{th} firm to the L-market is x_i, i = 1, 2 (Fig.1). The location of firms relative to each other is not specified, x_1 can be equal to, greater or less than x_2. Each firm chooses an optimal location which can be in one of the two markets or at a point on the line. The barbell model fits the reality well and can be used to examine the trade between two countries as well.

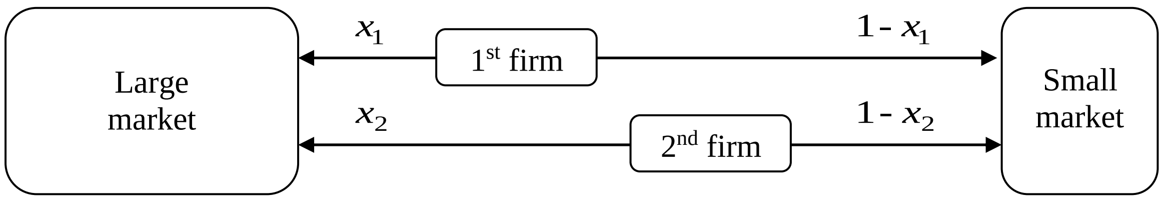

Fig.1 *The spatial duopoly model (barbell model)*
Source: author's development

The linear demand curves at each market (1):

$$p^L = 1 - k \cdot \sum_{i=1,2} q_i^L \Big/ \gamma, \quad p^S = 1 - k \cdot \sum_{i=1,2} q_i^S, \tag{1}$$

where p^L, p^S – the market prices, q_i^L, q_i^S – the quantities supplied of i^{th} firm, a minimum price, at which there is no demand (market potential), is equal to 1, k – the coefficient of price sensitivity, $\gamma \geq 1$ – the market size asymmetry coefficient (Fig.2).

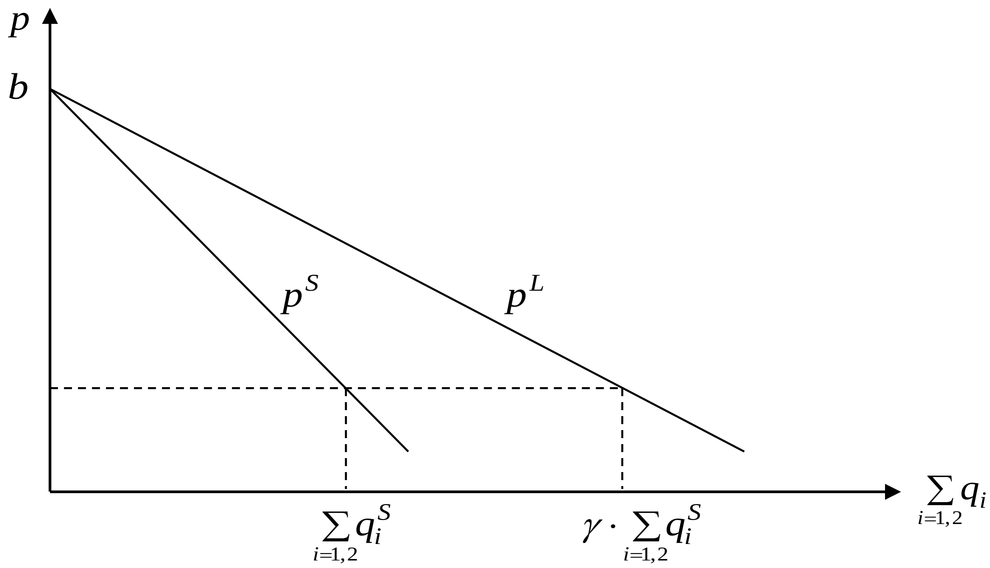

Fig.2 *The market size asymmetry*
Source: author's development

The competitive game consists of two stages. In the first stage, the firms simultaneously select their locations. In the second stage, at the given location decisions, the firms simultaneously choose their supplied quantities. The equilibrium of the model is solved by backward induction.

The profit of i^{th} firm is defined as the sum of its profits from both markets:

$$F_i = q_i^L \cdot \left(1 - k \cdot \left(q_i^L + q_j^L\right)/\gamma - t \cdot x_i\right) + q_i^S \cdot \left(1 - k \cdot \left(q_i^L + q_j^L\right) - t \cdot (1 - x_i)\right) \to \max_{x_i, q_i^L, q_i^S},$$
$$i = 1, 2, \; j = 3 - i. \tag{2}$$

2 THE COURNOT EQUILIBRIUM

Let us find the Cournot equilibrium. The analysis starts with the second stage. First, we find the optimal quantities of supplies. Solving the first-order conditions yields the reaction curves:

$$q_i^L = \gamma/(2 \cdot k) - q_j^L/2 - \gamma \cdot t \cdot x_i/(2 \cdot k), \quad q_i^S = 1/(2 \cdot k) - q_j^S/2 - t \cdot (1 - x_i)/(2 \cdot k),$$
$$q_j^L = \gamma/(2 \cdot k) - q_i^L/2 - \gamma \cdot t \cdot x_j/(2 \cdot k), \quad q_j^S = 1/(2 \cdot k) - q_i^S/2 - t \cdot \left(1 - x_j\right)/(2 \cdot k), \tag{3}$$

the second-order conditions (4):

$$\partial^2 F_i / \partial\left(q_i^L\right)^2 = -2 \cdot k/\gamma < 0, \; \partial^2 F_i / \partial\left(q_i^S\right)^2 = -2 \cdot k < 0, \; i = 1, 2, \; j = 3 - i. \tag{4}$$

Solving the system of equations (3) yields the Cournot equilibrium quantities of supplies:

$$\left(q_i^L\right)^C = \gamma \cdot \left(1 - 2 \cdot t \cdot x_i + t \cdot x_j\right) / (3 \cdot k), \quad \left(q_i^S\right)^C = \left(1 - 2 \cdot t \cdot (1 - x_i) + t \cdot (1 - x_i)\right) / (3 \cdot k). \quad (5)$$

The equations (5) show that the optimal volumes of the i^{th} firm increase when they are approaching the market and the competitors are distancing from the market (6):

$$\partial\left(q_i^L\right)^C / \partial x_i < 0, \; \partial\left(q_i^L\right)^C / \partial x_j > 0, \; \partial\left(q_i^S\right)^C / \partial x_i > 0, \; \partial\left(q_i^S\right)^C / \partial x_j < 0 \cdot \quad (6)$$

The optimal prices and profits are:

$$\left(p^L\right)^C = \left(1 + t \cdot \sum_{i=1,2} x_i\right) \Big/ 3, \quad \left(F_i^L\right)^C = \gamma \cdot \left(1 - 2 \cdot t \cdot x_i + t \cdot x_j\right)^2 / (9 \cdot k),$$

$$\left(p^S\right)^C = \left(1 + t \cdot \sum_{i=1,2} (1 - x_i)\right) \Big/ 3, \quad \left(F_i^S\right)^C = \left(1 - 2 \cdot t \cdot (1 - x_i) + t \cdot (1 - x_j)\right)^2 / (9 \cdot k), \quad (7)$$

$$F_i^C = \left(F_i^L\right)^C + \left(F_i^S\right)^C.$$

In the first stage each firm selects a profit-maximizing location at a given location of the competitor. Substitution of (7) into (2) and differentiation with respect to location gives:

$$\partial F_i^C / \partial x_i = -4 \cdot \gamma \cdot t \cdot \left(1 - 2 \cdot t \cdot x_i + t \cdot x_j\right) / (9 \cdot k) + 4 \cdot t \cdot \left(1 - 2 \cdot t \cdot (1 - x_i) + t \cdot (1 - x_j)\right) / (9 \cdot k) = 0, \quad (8)$$

the second-order condition:

$$\partial^2 F_i^C / \partial x_i^2 = 8 \cdot t^2 \cdot (\gamma + 1) / (9 \cdot k) > 0. \quad (9)$$

From the second-order condition (9) it follows that the profit function of i^{th} firm (2) is strictly convex with respect to location x_i. Thus, at equilibrium state firms will be located only in the markets, i.e. $x_i^e = 0$ or $x_i^e = 1$. Let us note that this result was first obtained in [5].

There are four possible locations for firms: in the same markets: $(x_1, x_2) = (0, 0), (1, 1)$, and in the different markets: $(x_1, x_2) = (0, 1), (1, 0)$.

Let us assume that there is no market size asymmetry, i.e. $\gamma = 1$. Then the profits of firms (7) in all cases are:

$$F_i^C(0, 0) = F_i^C(1, 1) = \left(t^2 + 2 \cdot (1 - t)\right) / (9 \cdot k),$$

$$F_i^C(0, 1) = F_i^C(1, 0) = \left(5 \cdot t^2 + 2 \cdot (1 - t)\right) / (9 \cdot k), \quad (10)$$

$$\Delta = F_i^C(0, 1) - F_i^C(0, 0) = F_i^C(1, 0) - F_i^C(1, 1) = 4 \cdot t^2 / (9 \cdot k).$$

It follows from (10) that in the absence of any asymmetries, firms will choose different markets: (0, 1) or (1, 0). This is the *competition effect*. The location of a competitor in the other market enhances the firm's market power. Let us note that with the growth of the transport tariff, the impact of the competition effect is intensified, $\partial\Delta/\partial t > 0$.

Let us explore how some asymmetries affect the location choice of firms. Let us consider three asymmetries: location asymmetry, market size asymmetry and Stackelberg information asymmetry.

The location asymmetry is given in the form of condition: $x_1 \le x_2$ (Fig.1). The location asymmetry leads to a single spatial equilibrium: (0, 1).

The market size asymmetry is given in the form of condition: $\gamma > 1$ (Fig. 2). Under market size asymmetry, the L-market begins to "attract" both firms to itself. This is the *market size effect*. Thus, under the action of two asymmetries, two spatial equilibrium states are possible: (0, 0) and (0, 1). Let us analyze a location decision of the 2nd firm:

$$F_2^C(0,0) - F_2^C(0,1) = 4 \cdot t \cdot (\gamma - \gamma \cdot t - 1)/(9 \cdot k) > (\le)\, 0,$$
$$\text{if} \quad t_2^C < (\ge)\, (\gamma - 1)/\gamma. \tag{11}$$

Expression (11) shows the minimal transport tariff at which the 2nd firm will remain in the S-market. With the reduction of the transportation tariff, $t^C < (\gamma - 1) / \gamma$, the L-market will "attract" the 2nd firm to itself and the agglomeration of firms will take place. The result (11) was first obtained in [6].

3 THE STACKELBERG EQUILIBRIA

The Stackelberg information asymmetry arises when one of the firms (leader) becomes aware of the competitor's strategy (follower). Under Stackelberg information asymmetry there are four possible equilibrium states (Table 1).

***Tab. 1** The Stackelberg equilibria*

Equilibrium / Markets	I	II	III	IV
Large	leader – 1st follower – 2nd	leader – 1st follower – 2nd	leader – 2nd follower – 1st	leader – 2nd follower – 1st
Small		leader – 2nd follower – 1st	leader – 1st follower – 2nd	

Source: author's development

Since the firm can be a leader in one market and a follower in the other one, let us consider the functions of firms' profits separately in each market. Assume that the i^{th} firm is a leader, and the j^{th} firm is a follower. Substituting the reaction curves of the j^{th} firm (3) into the profit functions of the i^{th} firm (2), we obtain:

$$F_i^L = 0{,}5 \cdot q_i^L \cdot \left(1 - k \cdot q_i^L / \gamma + t \cdot x_j - 2 \cdot t \cdot x_i\right) \to \max_{q_i^L}, \quad i = 1,2,\ j = 3 - i,$$
$$F_i^S = 0{,}5 \cdot q_i^S \cdot \left(1 - k \cdot q_i^S + t \cdot (1 - x_j) - 2 \cdot t \cdot (1 - x_i)\right) \to \max_{q_i^S}, \quad i = 1,2,\ j = 3 - i. \tag{12}$$

The firms’ profits in the Stackelberg equilibrium are derived by using standard procedure (Table 2).

***Tab. 2** The firms’ profits in the Stackelberg equilibrium*

	L-market	S-market
leader	$F_i^L = \gamma \cdot (1-2\cdot t\cdot x_i + t\cdot x_j)^2 / (8\cdot k),$	$F_i^S = (1-2\cdot t\cdot(1-x_i)+t\cdot(1-x_j))^2 / (8\cdot k),$
follower	$F_j^L = \gamma \cdot (1-3\cdot t\cdot x_j + 2\cdot t\cdot x_i)^2 / (16\cdot k),$	$F_j^S = (1-3\cdot t\cdot(1-x_j)+2\cdot t\cdot(1-x_i))^2 / (16\cdot k),$
	$i=1,2,\ j=3-i.$	$i=1,2,\ j=3-i.$

Source: author's development

We will then analyze the firms’ optimal location under the impact of three asymmetries.

3.1 Equilibrium I: 1st firm – leader, 2nd firm – follower in both markets

The firms’ equilibrium profits are (14):

$$F_1^I = \gamma \cdot (1-2\cdot t\cdot x_1 + t\cdot x_2)^2 / (8\cdot k) + (1-2\cdot t\cdot(1-x_1)+t\cdot(1-x_2))^2 / (8\cdot k),$$
$$F_2^I = \gamma \cdot (1-3\cdot t\cdot x_2 + 2\cdot t\cdot x_1)^2 / (16\cdot k) + (1-3\cdot t\cdot(1-x_2)+2\cdot t\cdot(1-x_1))^2 / (16\cdot k). \quad (14)$$

If the 2nd firm chooses the L-market, then the 1st firm will be located in the L-market because of the location asymmetry. Let us find a location decision of 1st firm if the 2nd firm is located in the S-market (15):

$$F_1^I(0,1) - F_1^I(1,1) = t\cdot(\gamma-1+t)/(2\cdot k) > 0\cdot \quad (15)$$

Thus, in the Equilibrium I the 1st firm will always be located in the L-market. Let us analyze a location decision of the 2nd firm:

$$F_2(0,0) - F_2(0,1) = 3\cdot t\cdot(2\cdot(\gamma-1) - t\cdot(3\cdot\gamma+1))/(16\cdot k) > (\le)\, 0,$$
$$\text{if } t_2^I < (\ge)\, 2\cdot(\gamma-1)/(3\cdot\gamma+1)\cdot \quad (16)$$

It follows from (16) that the level of the minimal tariff has decreased (17):

$$t_2^I - t_2^C = 2\cdot(\gamma-1)/(3\cdot\gamma+1) - (\gamma-1)/\gamma = -(\gamma^2-1)/(\gamma\cdot(3\cdot\gamma+1)) < 0. \quad (17)$$

Thus, the follower's position in both markets strengthens the competition effect for the 2nd firm and weakens the market size effect. This is due to the impact of the *information asymmetry effect*.

3.2 Equilibrium II: 1st firm – leader, 2nd firm – follower in the L-market; 2nd firm – leader, 1st firm– follower in the S-market

The firms' equilibrium profits are (18):

$$F_1^{II} = \gamma \cdot (1 - 2 \cdot t \cdot x_1 + t \cdot x_2)^2 / (8 \cdot k) + (1 - 3 \cdot t \cdot (1 - x_1) + 2 \cdot t \cdot (1 - x_2))^2 / (16 \cdot k),$$
$$F_2^{II} = \gamma \cdot (1 - 3 \cdot t \cdot x_2 + 2 \cdot t \cdot x_1)^2 / (16 \cdot k) + (1 - 2 \cdot t \cdot (1 - x_2) + t \cdot (1 - x_1))^2 / (8 \cdot k). \quad (18)$$

If the 2nd firm chooses the L-market, then the 1st firm will be located in the L-market because of the location asymmetry. Let us find a location decision of the 1st firm if the 2nd firm is located in the S-market (19):

$$F_1^{II}(0,1) - F_1^{II}(1,1) = t \cdot (8 \cdot \gamma - 6 + 9 \cdot t)/(16 \cdot k) > 0 \cdot \quad (19)$$

Thus, in the Equilibrium II the 1st firm will always be located in the L-market.
Let us analyze a location decision of the 2nd firm:

$$F_2^{II}(0,0) - F_2^{II}(0,1) = t \cdot (6 \cdot \gamma - 8 - 9 \cdot t \cdot \gamma)/(16 \cdot k) > (\le)\, 0,$$
$$\text{if } t_2^{II} < (\ge)\, (6 \cdot \gamma - 8)/(9 \cdot \gamma). \quad (20)$$

From (20) it follows that the level of the minimal tariff has further decreased (21):

$$t_2^{II} - t_2^{I} = (6 \cdot \gamma - 8)/(9 \cdot \gamma) - 2 \cdot (\gamma - 1)/(3 \cdot \gamma + 1) = -8/(9 \cdot \gamma \cdot (3 \cdot \gamma + 1)) < 0. \quad (21)$$

Thus, the leader's position in the S-market has further strengthened the competition effect for the 2nd firm. From (21) it also follows that the market size effect is possible only at $\gamma > 4/3$. At $\gamma \le 4/3$, 2nd firm will always be located in the S-market.

3.3 Equilibrium III: 2nd firm – leader, 1st firm – follower in the L-market; 1st firm – leader, 2nd firm – follower in the S-market

The firms' equilibrium profits are (22):

$$F_1^{III} = \gamma \cdot (1 - 3 \cdot t \cdot x_1 + 2 \cdot t \cdot x_2)^2 / (16 \cdot k) + (1 - 2 \cdot t \cdot (1 - x_1) + t \cdot (1 - x_2))^2 / (8 \cdot k),$$
$$F_2^{III} = \gamma \cdot (1 - 2 \cdot t \cdot x_2 + t \cdot x_1)^2 / (8 \cdot k) + (1 - 3 \cdot t \cdot (1 - x_2) + 2 \cdot t \cdot (1 - x_1))^2 / (16 \cdot k). \quad (22)$$

If the 2nd firm chooses the L-market, then the 1st firm will be located in the L-market because of the location asymmetry. Let us find a location decision of the 1st firm if the 2nd firm is located in the S-market:

$$F_1^{III}(1,1)-F_1^{III}(0,1)=t\cdot(8-6\cdot\gamma-t\cdot(3\cdot\gamma+8))/(16\cdot k)>(\le)\,0,$$
$$\text{if } t_1^{III}<(\ge)\,(8-6\cdot\gamma)/(3\cdot\gamma+8). \quad (23)$$

It follows from (23) that for $\gamma < 4/3$ there is transport tariff at which the 1st firm will choose the S-market. At $\gamma \ge 4/3$, 1st firm will always be located in the L-market.

If the 1st firm chooses the S-market, then the 2nd firm will be located in the S-market because of the location asymmetry. Let us find a location decision of the 2nd firm if the 1st firm is located in the L-market (24):

$$F_2^{III}(0,0)-F_2^{III}(0,1)=t\cdot(8\cdot\gamma-6-t\cdot(8\cdot\gamma+3))/(16\cdot k)>(\le)\,0,$$
$$\text{if } t_2^{III}<(\ge)\,(8\cdot\gamma-6)/(8\cdot\gamma+3). \quad (24)$$

It is found that in the equilibrium III the location of firms significantly depends on the level of the market size asymmetry. At $\gamma \ge 4/3$, agglomeration is possible in the L-market, and at $\gamma < 4/3$, agglomeration is possible in the S-market. The final location of firms will depend on who makes the first move in the game. Thus, in the equilibrium III, three spatial equilibrium states are possible: (0, 0), (0, 1) and (1, 1).

From (24) it follows that the level of the minimal tariff has increased (25):

$$t_2^{III}-t_2^{I}=(8\cdot\gamma-6)/(8\cdot\gamma-3)-2\cdot(\gamma-1)/(3\cdot\gamma+1)=\left(8\cdot\gamma^2+12\cdot(\gamma-1)\right)/\left((8\cdot\gamma-3)\cdot(3\cdot\gamma+1)\right)>0. \quad (25)$$

Thus, the leader's position in the L-market has decreased the impact of the competition effect for the 2nd firm.

3.4 Equilibrium IV: 1st firm – follower, 2nd firm – leader in both markets

The firms' equilibrium profits are (26):

$$F_1^{IV}=\gamma\cdot(1-3\cdot t\cdot x_1+2\cdot t\cdot x_2)^2/(16\cdot k)+(1-3\cdot t\cdot(1-x_1)+2\cdot t\cdot(1-x_2))^2/(16\cdot k),$$
$$F_2^{IV}=\gamma\cdot(1-2\cdot t\cdot x_2+t\cdot x_1)^2/(8\cdot k)+(1-2\cdot t\cdot(1-x_2)+t\cdot(1-x_1))^2/(8\cdot k). \quad (26)$$

If the 2nd firm chooses the L-market, then the 1st firm will be located in the L-market because of the location asymmetry. Let us find a location decision of 1st firm if the 2nd firm is located in the S-market (27):

$$F_1^{IV}(0,1)-F_1^{IV}(1,1)=t\cdot(6\cdot(\gamma-1)+3\cdot t\cdot(\gamma+3))/(16\cdot k)>0. \quad (27)$$

Thus, in the Equilibrium IV the 1st firm will always be located in the L-market. Let us analyze a location decision of the 2nd firm:

$$F_2^{IV}(0,0)-F_2^{IV}(0,1)=t\cdot(\gamma-t\cdot\gamma-1)/(8\cdot k)>(\le)\,0,$$
$$\text{if } t_2^{IV}<(\ge)\,(\gamma-1)/\gamma. \quad (28)$$

Let us note that solution (28) coincides with the location solution of the 2nd firm in the Cournot equilibrium (11), $t_2^{IV} = t_2^C$. Thus, when the 2nd firm is a leader in both markets, the information asymmetry effect does not affect the optimal location solutions of the firms.

Thus, the optimal locations are determined by three effects – the market size effect, the competition effect and the information asymmetry effect. The market size effect attracts firms to the large market, the competition effect pushes the firms away from each other, and the information asymmetry effect attracts firms to the market where they are leaders.

Possibility of the firms' agglomeration in the small market extends the results derived by Liang et al. (2006), who considered two effects: the market size effect and the competition effect.

4 CONCLUSIONS

This paper has developed a barbell model under Stackelberg information asymmetry to analyze the firms' equilibrium locations. A comparative analysis of the location choice of the firms is carried out.

In the Cournot equilibrium the optimal firms' locations are determined by two factors: the market size effect and the competition effect. The market size effect moves firms to the large market while the competition effect pushes the firms away from each other. When one market is sufficiently larger than the other one or transport tariff is sufficiently low, the market size effect dominates the competition effect and both firms are located in the large market.

In the Stackelberg equilibrium the optimal firms' locations are determined by an additional factor – the information asymmetry. Under the information asymmetry one of the firms has information about a competitor's strategy. It is found that an increase of the transportation tariff contributes to the dispersion of firms, and the growth of the size market asymmetry contributes to the agglomeration of firms in the large market. It is obtained that under certain conditions the information asymmetry effect dominates the market size effect and both firms agglomerate in the small market. In this case the optimal location choice of firms depends on who makes the first move in the game.

In further research, the barbell model can be generalized to the case of a set of markets and firms. It would be interesting to investigate how an optimal location of firms will depend on other asymmetry types, e.g. of quality, costs, etc.

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