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STANDARD DEVIATION OF THE MEASURING POINTS COORDINATES AT DIRECTIONAL BOREHOLES BY USING AVERAGE ANGLE METHOD

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Abstract: Directional wells must be made in conformity with the project. During their development, measurements made within the well bore to determine inclination and azimuth are necessary. The results of those measurements and known depths measured from the well head enable computation of spatial coordinates of the measuring points. All measurement results are burdened by errors, and pursuant to the Error Propagation Law their influence is also reflected to the accuracy of the coordinates of measurement points. Errors of the coordinates of measuring points, apart from measurement errors, also depend on functional connections connecting measured and unknown values. There are several methods for computing the spatial position of the well bore applied in practical work. This paper, on the example of directional development well X-1 in Panonian basin, shows computation of coordinates of the measuring points by applying Average Angle Method and their standard deviation.

1. Introduction

Technique of directional drilling is used for many years for the purpose of reaching of deep structures beneath the inaccessible or hardly accessible locations. Economic reasons and the requirements of environmental protection have increased the number of directional and horizontal wells made during last years. There are many factors that need to be taken into account when deciding on the optimal section for given well and they include: location, thickness and slope of target interval; formation characteristics and he need for casing; coefficient of friction of the formation; degree of geological uniformity; set-up method; necessary weight on bit; local availability of equipment; plant capacity.

Actual depth of target i.e. depth of deposit and deflection from basic borehole or from surface location have the largest impact on the formed section and slope of the layer represents the factor that determines the final borehole inclination.During the drilling of directional wells, it is necessary to perform measurements, by which the spatial coordinates of measuring points within the borehole are to be determined. By linking the adjacent measuring points, the trajectory of the borehole, which serves for the analysis of the derived condition in relation to the projected axis, is obtained.

Instruments used to perform those measurements are of various types and features, and they are divided into two large groups: single-shot and multi-shot instruments. Basic difference is that at the first group of instruments, after the measurements have been carried out, the equipment

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is to be taken out of the borehole, so it is necessary to take it down again and so the process goes on cyclically, while at the second type of instruments, measurements are being carried out continuously.

Regardless the type of instruments, in order to determine spatial coordinates of instruments, namely of the measuring points in the borehole, it is necessary to measure the angle of inclination and azimuth. Based on those measurements and on the known depth of borehole from the borehole mouth, the coordinates of measuring points are to be calculated.

Measuring uncertainty of instruments, which are used to measure the inclination and azimuth, causes that all measurement results are burdened with errors. Pursuant to the Law of the propagation of errors, such incorrect measurement results cause that derived – calculated parameters are incorrect as well. This means that the spatial coordinates of measuring points in the borehole are to be calculated with certain error as well.

The accuracy of measurement results is influenced by the errors of instruments as well as a number of other errors, but also the functional relations – mathematical equation that connects parameters that are being calculated with measurement results.

In case of calculation of spatial coordinates of measuring points in directional boreholes, the most common in practical work are: tangential method, balanced tangential method, average angle method, radius of curvature method and minimum curvature method.

2. Directional development borehole X-1

Inclined directional development well X-1 is located on basis of structural contour map on deposit roof "M+Tg" (middle Miocene sandstones). The entrance to the deposit M+Tg is at the vertical depth of 1948 m. Horizontal distance from the wellhead to the entrance to the deposit M+Tg is 381 m, and azimuth is 166º.

The deflection from the vertical hole of the well KOP is going to be at the depth of 1500 m. Because of the planning of future stimulation works (fracturing) and setting up of the borehole, the trajectory of well is to be made as "S" profile.

The deposit M-Tg within the oil and gas field Kikinda varoš is situated immediately adjacent to the town of Kikinda and partially also beneath the town itself. Based on geological interpretation, production characteristics of the nearest wells and based on the interpretation of three-dimensional seismology, the south-east part of the deposit is determined for drilling of new wells.

Geological formation containing hydrocarbons is Sarmat represented by sandstones and conglomerate sandstones.

3. Calculation of coordinates of measuring points by using the Average Angle Method

This method is based on calculation of arithmetic means of measured angles of inclination and azimuths on two measuring points. In this way, the borehole trajectory is obtained, whose length equals the actual length between measuring points. In situation when the distance between measuring points is not very large when compared to the borehole trajectory curvature, this method provides simple however sufficiently precise calculation of the borehole trajectory.

Coordinate differences within the horizontal plane between the adjacent measuring points obtained by average angle method, are to be calculated according to the following equations:

$$\Delta N = \Delta MD \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \cdot \cos\left(\frac{\alpha_1 + \alpha_2}{2}\right) \tag{1}$$

$$\Delta E = \Delta MD \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \cdot \sin\left(\frac{\alpha_1 + \alpha_2}{2}\right) \tag{2}$$

and within the vertical plane:

$$\Delta TVD = \Delta MD \cdot \cos\left(\frac{i_1 + i_2}{2}\right) \tag{3}$$

Coordinates of measuring points are obtained when corresponding coordinate differences are added to preceding points, namely:

$$N_i = N_{i-1} + \Delta N_{i-1}^i; \quad E_i = E_{i-1} + \Delta E_{i-1}^i; \quad Z_i = Z_{i-1} + \Delta TVD_{i-1}^i \tag{4}$$

Coordinates of measuring points within the inclined directional development well X-1 computed upon the Average Angle Method are shown in Table 1. Calculation includes only measuring points within directional borehole from measuring point 0 located at the depth of 1500 m up to the measuring point number 32 located in the deposit M+Tg.

***Tab. 1** Coordinates of measuring points.*

Point	MD [m]	*i* [°]	α [°]	ΔN [m]	ΔE [m]	ΔTVD [m]	*N* [m]	*E* [m]	*Z* [m]
0	1500,00	0,00	220				**7295,37**	**8792,40**	**1500,00**
1	1546,00	2,75	219	-0,85	-0,70	45,99	**7294,52**	**8791,70**	**1545,99**
2	1560,00	2,25	160	-0,60	-0,10	13,99	**7293,92**	**8791,60**	**1559,97**
3	1570,00	2,75	140	-0,38	0,22	9,99	**7293,54**	**8791,82**	**1569,96**
4	1579,00	5,75	130	-0,47	0,47	8,98	**7293,07**	**8792,29**	**1578,94**
5	1598,00	10,00	176	-2,32	1,18	18,82	**7290,75**	**8793,47**	**1597,76**
6	1608,00	13,75	180	-2,06	0,07	9,79	**7288,69**	**8793,54**	**1607,55**
7	1628,00	24,00	170	-6,45	0,56	18,92	**7282,25**	**8794,10**	**1626,47**
8	1647,00	29,00	168	-8,32	1,62	17,00	**7273,92**	**8795,72**	**1643,47**
9	1667,00	29,25	158	-9,31	2,85	17,47	**7264,61**	**8798,57**	**1660,95**
10	1677,00	31,00	153	-4,57	2,08	8,65	**7260,05**	**8800,65**	**1669,59**
11	1696,00	37,25	151	-9,41	5,00	15,73	**7250,64**	**8805,65**	**1685,32**
12	1706,00	41,25	149	-5,48	3,16	7,74	**7245,16**	**8808,82**	**1693,07**
13	1723,00	48,00	144	-9,96	6,59	12,10	**7235,20**	**8815,41**	**1705,17**
14	1781,00	56,25	145	-37,27	26,59	35,61	**7197,93**	**8841,99**	**1740,78**
15	1839,35	59,00	150	-41,56	26,48	31,24	**7156,36**	**8868,47**	**1772,02**
16	1896,00	59,25	155	-43,13	22,45	29,07	**7113,24**	**8890,92**	**1801,09**
17	1916,00	62,25	146	-15,41	8,72	9,31	**7097,83**	**8899,64**	**1810,40**
18	1926,00	65,00	145	-7,48	5,14	4,21	**7090,35**	**8904,78**	**1814,61**
19	1935,00	61,75	143	-6,51	4,73	4,03	**7083,84**	**8909,51**	**1818,64**
20	1944,00	59,25	144	-6,30	4,66	4,43	**7077,55**	**8914,17**	**1823,07**
21	1953,00	57,00	145	-6,22	4,44	4,75	**7071,33**	**8918,60**	**1827,83**
22	1963,00	54,00	146	-6,79	4,67	5,66	**7064,53**	**8923,27**	**1833,49**
23	1972,00	51,00	148	-5,99	3,89	5,48	**7058,55**	**8927,16**	**1838,97**
24	1981,00	48,25	150	-5,88	3,53	5,83	**7052,67**	**8930,69**	**1844,80**
25	2001,00	47,00	156	-13,16	6,71	13,48	**7039,50**	**8937,40**	**1858,28**
26	2030,00	44,00	158	-19,04	8,08	20,33	**7020,46**	**8945,48**	**1878,61**
27	2058,00	41,50	162	-17,86	6,50	20,56	**7002,60**	**8951,98**	**1899,17**
28	2089,00	41,00	167	-19,70	5,46	23,31	**6982,91**	**8957,45**	**1922,47**
29	2116,00	42,75	168	-17,60	3,90	20,10	**6965,31**	**8961,35**	**1942,58**
30	2122,00	43,00	167	-3,99	0,88	4,40	**6961,33**	**8962,23**	**1946,97**
31	2155,00	44,75	166	-22,24	5,34	23,79	**6939,09**	**8967,57**	**1970,76**
32	2162,00	45,00	165	-4,78	1,24	4,96	**6934,30**	**8968,81**	**1975,72**

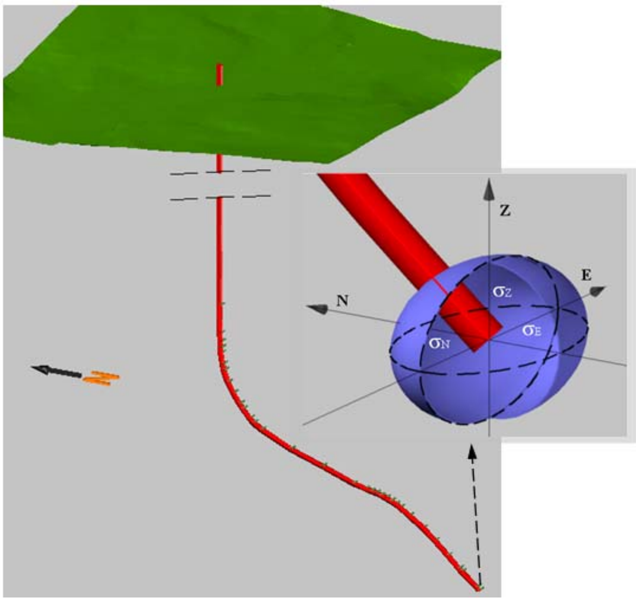

***Fig. 1** Spatial location of the well X-1 with the ellipsoid of errors of the measuring point 32.*

4. Standard deviation of measuring points coordinates

Standard deviation of measuring points coordinates calculated by using the Average Angle Method is obtained when partial derivatives upon measured parameters i_1, α_1 in the preceding and i_2, α_2 in the actual measuring points are calculated from the equations (1, 2 and 3) and then multiplied by standard deviations σ_{i_1}, σ_{α_1}, respectively σ_{i_2}, σ_{α_2} in the equation that defines the Law of the propagation of errors.

Partial derivative of the equation (1) upon the measured inclination i_1 is:

$$k_{i_1} = \frac{\partial \Delta N}{\partial i_1} = \frac{\Delta MD}{2} \cdot \cos\left(\frac{i_1 + i_2}{2}\right) \cdot \cos\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (5)$$

upon measured azimuth α_1:

$$k_{\alpha_1} = \frac{\partial \Delta N}{\partial \alpha_1} = -\frac{\Delta MD}{2} \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \cdot \sin\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (6)$$

upon inclination i_2:

$$k_{i_2} = \frac{\partial \Delta N}{\partial i_2} = \frac{\Delta MD}{2} \cdot \cos\left(\frac{i_1 + i_2}{2}\right) \cdot \cos\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (7)$$

and upon azimuth α_2:

$$k_{\alpha_2} = \frac{\partial \Delta N}{\partial \alpha_2} = -\frac{\Delta MD}{2} \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \cdot \sin\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (8)$$

Standard deviation of the coordinate difference ΔN is:

$$\sigma_{\Delta N} = \sqrt{k_{i_1}^2 \cdot \sigma_{i_1}^2 + k_{\alpha_1}^2 \cdot \sigma_{\alpha_1}^2 + k_{i_2}^2 \cdot \sigma_{i_2}^2 + k_{\alpha_2}^2 \cdot \sigma_{\alpha_2}^2} \quad (9)$$

Partial derivative of the equation (2) upon measured inclinations and azimuths is:

$$k_{i_1} = \frac{\partial \Delta E}{\partial i_1} = \frac{\Delta MD}{2} \cdot \cos\left(\frac{i_1 + i_2}{2}\right) \cdot \sin\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (10)$$

$$k_{\alpha_1} = \frac{\partial \Delta E}{\partial \alpha_1} = \frac{\Delta MD}{2} \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \cdot \cos\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (11)$$

$$k_{i_2} = \frac{\partial \Delta E}{\partial i_2} = \frac{\Delta MD}{2} \cdot \cos\left(\frac{i_1 + i_2}{2}\right) \cdot \sin\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (12)$$

$$k_{\alpha_2} = \frac{\partial \Delta E}{\partial \alpha_2} = \frac{\Delta MD}{2} \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \cdot \cos\left(\frac{\alpha_1 + \alpha_2}{2}\right) \quad (13)$$

and standard deviation of the coordinate difference ΔE is:

$$\sigma_{\Delta E} = \sqrt{k_{i_1}^2 \cdot \sigma_{i_1}^2 + k_{\alpha_1}^2 \cdot \sigma_{\alpha_1}^2 + k_{i_2}^2 \cdot \sigma_{i_2}^2 + k_{\alpha_2}^2 \cdot \sigma_{\alpha_2}^2} \quad (14)$$

Partial derivatives of the equation (3) upon measured inclinations are:

$$k_{i_1} = \frac{\partial \Delta TVD}{\partial i_1} = -\frac{\Delta MD}{2} \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \quad (15)$$

$$k_{i_2} = \frac{\partial \Delta TVD}{\partial i_2} = -\frac{\Delta MD}{2} \cdot \sin\left(\frac{i_1 + i_2}{2}\right) \tag{16}$$

and standard deviation of coordinate difference ΔTVD is:

$$\sigma_{\Delta TVD} = \sqrt{k_{i_1}^2 \cdot \sigma_{i_1}^2 + k_{i_2}^2 \cdot \sigma_{i_2}^2} \tag{17}$$

Standard deviation of measuring points coordinates is, on the basis of the standard deviations of coordinates of the preceding point and known standard deviations of corresponding coordinate differences, is calculated upon the equations:

$$\sigma_{N_i} = \sqrt{\sigma_{N_{i-1}}{}^2 + \sigma_{\Delta N_{i-1}^i}{}^2}; \quad \sigma_{E_i} = \sqrt{\sigma_{E_{i-1}}{}^2 + \sigma_{\Delta E_{i-1}^i}{}^2}; \quad \sigma_{Z_i} = \sqrt{\sigma_{Z_{i-1}}{}^2 + \sigma_{\Delta TVD_{i-1}^i}{}^2} \tag{18}$$

Standard deviations of measuring points' coordinates in the borehole X-1 are shown in Table 2. Thereby, for the measuring point 0, where the vertical section of the borehole ends and the deflection starts, it is adopted that $\sigma_N=\sigma_E=\sigma_Z=0$, or to be precise, that there are no deviations at this point. Calculated deviations for the measuring point 32, in this case, are obtained in relation to the point 0 and they represent the product of measuring uncertainty of the measuring equipment that has been used and the computation method applied.

Tab. 2 *Standard deviation of the measuring points' location.*

Point	σ_i [°]	σ_α [°]	$\sigma_{\Delta N}$ [m]	$\sigma_{\Delta E}$ [m]	$\sigma_{\Delta TVD}$ [m]	σ_N [m]	σ_E [m]	σ_Z [m]
0	0,3	0,5				**0**	**0**	**0**
			0,1314	0,1084	0,0041			
1	0,3	0,5				**0,1314**	**0,1084**	**0,0041**
			0,0511	0,0093	0,0023			
2	0,3	0,5				**0,1410**	**0,1088**	**0,0047**
			0,0321	0,0186	0,0016			
3	0,3	0,5				**0,1446**	**0,1104**	**0,0049**
			0,0237	0,0237	0,0025			
4	0,3	0,5				**0,1465**	**0,1129**	**0,0055**
			0,0625	0,0347	0,0096			
5	0,3	0,5				**0,1593**	**0,1181**	**0,0111**
			0,0362	0,0128	0,0076			
6	0,3	0,5				**0,1634**	**0,1188**	**0,0135**
			0,0699	0,0402	0,0240			
7	0,3	0,5				**0,1777**	**0,1255**	**0,0275**
			0,0626	0,0527	0,0314			
8	0,3	0,5				**0,1884**	**0,1361**	**0,0417**
			0,0643	0,0605	0,0360			
9	0,3	0,5				**0,1991**	**0,1489**	**0,0551**
			0,0318	0,0312	0,0186			
10	0,3	0,5				**0,2016**	**0,1521**	**0,0582**
			0,0600	0,0642	0,0395			
11	0,3	0,5				**0,2103**	**0,1651**	**0,0703**
			0,0316	0,0367	0,0234			
12	0,3	0,5				**0,2127**	**0,1692**	**0,0741**
			0,0552	0,0662	0,0442			
13	0,3	0,5				**0,2198**	**0,1817**	**0,0863**
			0,1960	0,2424	0,1695			
14	0,3	0,5				**0,2945**	**0,3029**	**0,1902**
			0,1903	0,2639	0,1825			
15	0,3	0,5				**0,3506**	**0,4017**	**0,2636**
			0,1682	0,2707	0,1800			
16	0,3	0,5				**0,3889**	**0,4845**	**0,3192**
			0,0616	0,0966	0,0655			
17	0,3	0,5				**0,3938**	**0,4940**	**0,3258**
			0,0342	0,0470	0,0336			
18	0,3	0,5				**0,3952**	**0,4962**	**0,3276**
			0,0316	0,0411	0,0298			
19	0,3	0,5				**0,3965**	**0,4979**	**0,3289**
			0,0316	0,0401	0,0290			
20	0,3	0,5				**0,3978**	**0,4995**	**0,3302**
			0,0309	0,0397	0,0283			
21	0,3	0,5				**0,3990**	**0,5011**	**0,3314**
			0,0336	0,0436	0,0305			
22	0,3	0,5				**0,4004**	**0,5030**	**0,3328**
			0,0294	0,0386	0,0264			
23	0,3	0,5				**0,4014**	**0,5045**	**0,3338**
			0,0286	0,0379	0,0254			
24	0,3	0,5				**0,4025**	**0,5059**	**0,3348**
			0,0608	0,0843	0,0547			
25	0,3	0,5				**0,4070**	**0,5129**	**0,3393**
			0,0854	0,1211	0,0766			
26	0,3	0,5				**0,4159**	**0,5270**	**0,3478**
			0,0820	0,1132	0,0704			
27	0,3	0,5				**0,4239**	**0,5390**	**0,3548**
			0,0897	0,1237	0,0757			
28	0,3	0,5				**0,4333**	**0,5530**	**0,3628**
			0,0766	0,1098	0,0667			
29	0,3	0,5				**0,4400**	**0,5638**	**0,3689**
			0,0168	0,0248	0,0151			
30	0,3	0,5				**0,4403**	**0,5644**	**0,3692**
			0,0918	0,1388	0,0847			
31	0,3	0,5				**0,4498**	**0,5812**	**0,3788**
			0,0193	0,0299	0,0183			
32	0,3	0,5				**0,4502**	**0,5819**	**0,3792**

5. Conclusion

Presented Average Angle Method represents a simple and practical method for determination of spatial coordinates of the borehole, whose results fulfill the criteria of the required accuracy for the development of exploitation wells. By applying this method, in the example in case, spatial deviation at the end point of the borehole (point 32, Table 2) of ± 0.83 m was obtained, which for the length of the inclined channel of 662 m amounts 0.13 %. Considering the fact that the value of deviation has no bearing to the quality of the spatial positioning of the well bore, the Average Angle Method is applicable while developing inclined directional wells in order to reach oil and gas deposits at the depths larger than the depth shown in the paper.

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