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INTEGRATED TIMETABLE AND SCHEDULING OPTIMIZATION WITH MULTIOBJECTIVE EVOLUTIONARY ALGORITHM

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Abstract: *This paper is discussing the integration of timetabling and vehicle scheduling stage of the transportation planning process with implementation of evolutionary multiobjective genetic algorithm (NSGA-II). Test case in public transport with minimization of the transfer time of the passengers in transfer node along with minimization of the number of vehicles needed to operate such timetable is presented. Other applications in the freight transport field are also discussed. Developed solution proposes that it is able to optimize conflicting objectives of passengers or customers and transportation company simultaneously.*
Key words: *multiobjective optimization, genetic algorithm, timetable, vehicle scheduling*

1 INTRODUCTION

Planning process of transportation service typically consists of several distinctive stages. For example, planning process of public transport service has 4 major parts:
network design (routing), timetabling, vehicle scheduling, crew scheduling and rostering. Similar pattern with separate stages can be observed also in freight transportation planning.
In real world sized transportation systems, each planning stage is a complex task on its own. Therefore, the stages are executed sequentially, usually in direction from the strategical level to the operational level. The stages are interconnected and the output of one stage is an input of the next stage. Moreover, the stages have different, often conflicting objectives. While some stages of the planning process focus more on the customer (passenger in case of public transport), other tend to concentrate more on the transport company's view. Typical example can be maximizing transport service quality mainly determined by network design and timetabling in public transport, and simultaneously minimization of cost related to vehicle and crew scheduling. One can see, that ideally the whole planning process should be optimized to a global optimality instead of local objectives of separate planning stages. In the transportation research field, there are few works integrating timetabling and scheduling. Examples include Periodic Event Scheduling problem for railway with partial integration of other planning aspects in [1], an approach using iterated local search in [2] and [3] .
The objective of this paper is to discuss integrated planning approach employing multiobjective evolutionary algorithm. Rest of this paper is organized as follows: in next

section, basic theory on multiobjective optimization and multiobjective genetic algorithm NSGA-II is presented. Thereafter, application of this approach both in public and freight transportation is discussed.

2 MULTIOBJECTIVE OPTIMIZATION AND EVOLUTIONARY ALGORITHMS

In contrast to single-objective optimization, multiobjective optimization deals with several objective functions. General multiobjective optimization problem can be defined as follows [4]:

Minimize $f(x) = [f_1(x), f_2(x), \dots, f_k(x)]$
Subject to:
$g_j(x) \leq 0, \quad j = 1, 2, \dots, m,$
$h_l(x) = 0, \quad l = 1, 2, \dots, e.$

where k is the number of objective functions, m is the number of inequality constraints, and e is the number of equality constraints. A solution $x \in R^n$ is a vector of n decision variables in the solution space **X**: $x = \{x_1, x_2, ..., x_n\}$. The objective is to find a vector x^* that minimizes a given set of K objective functions $f(x^*) = \{f_1(x^*), \dots, f_K(x^*)\}$.

Usually, there is not just a single solution to the multiobjective optimization problem, but more solutions can be optimal. Then the goal of multiobjective optimization is to find possibly all solutions each of which minimizes the objective functions at an acceptable level. The most used definition of optimality, that is when the solution is defined as optimal, is Pareto optimality.

2.1 Concept of Pareto optimality

A solution $x_1 \in X$ is Pareto optimal if there is no other solution $x_2 \in X$ such that $f(x_2) \leq f(x_1)$, and $f_i(x_2) < f_i(x_1)$ for at least one function [4].

This Pareto optimal solution in the objective space **Z** is called non-dominated. A Pareto-optimal solution can not be improved in any objective without worsening in at least one other objective. The all Pareto optimal solutions in solution space **X** constitute the Pareto optimal set.

The corresponding values of the objective functions of the Pareto optimal solutions in the objective space constitute Pareto front.

Assume two objective functions, f_1, f_2. Then Fig. 1 shows the Pareto front which is highlighted in objective space. For example, solution *i* is dominated by solutions *c*, *d* and *e*. Solutions *f*, *g* and *h* are dominated by only a single solution *a*, *b*, *b*, respectively. The Pareto optimality concept means that all Pareto optimal solutions are equally optimal, i.e. we can not say solution *b* is better than solution *a*, for example.

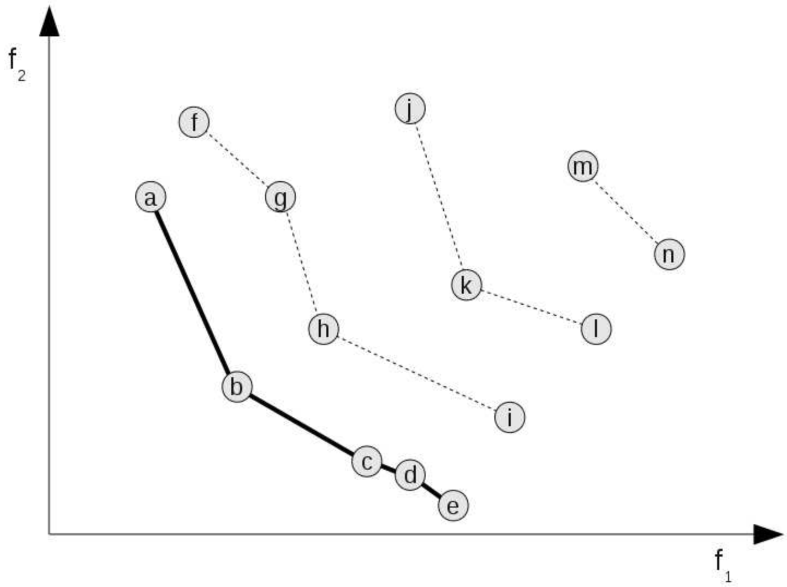

Fig. 1 *Pareto front in objective space*

There are several approaches how to tackle multiobjective optimization problem. Some of the most well known include Weighted sum method, Lexicographic method, in which the objective functions are arranged in order of importance, Weighted product method, or Multiobjective evolutionary algorithms. Introduction an review of multiobjective evolutionary optimization can be found in [5] and [6].

2.2 Fast Non-dominated Sorting Genetic Algorithm (NSGA-II)

One of the evolutionary algorithms adapted to multiobjective optimization is well tested and computationally efficient Fast Non-dominated Sorting Genetic Algorithm NSGA-II by Deb [7].

Evolutionary algorithms are broad category of optimization algorithms (including Genetic algorithm and Differential evolution) which use a population based search. The population iteratively evolves with each new generation. Traditional evolutionary algorithms are single-objective in which the fittest individual (with highest objective function value) represents the single suboptimal solution. The fact that evolutionary algorithms work with multiple solutions at time make them very suitable for multiobjective optimization.

NSGA-II evaluates individual solutions by dominance rank and crowding distance instead of objective function value. Crowding distance promotes search of solutions uniformly spread along the Pareto front. The crowding distance is used in following manner (Fig. 2):

Step 1: The population is ranked by dominance rule and non-dominated fronts $F_1, F_2,...,F_R$ are identified. For each front F_j, $j = 1, \ldots, R$ repeat Steps 2 and 3.

Step 2: The solutions in front F_j are sorted in ascending order. The sorting is repeated for each objective function *f*. Let $l = |F_j|$ and $x_{i,k}$ represent the *i*-th solution in the sorted list with respect to the objective function f_k.. Assign $cd_k(x_{1,k}) = \infty$ and $cd_k(x_{l,k}) = \infty$, and then assign for $i = 2, \ldots, l-1$:

$$cd_k(x_{i,k}) = \frac{f_k(x_{i+1,k}) - f_k(x_{i-1,k})}{f_k^{\max} - f_k^{\min}}$$

Step 3: To compute the total crowding distance *cd(x)* of a solution *x*, the solution's crowding distances with respect to each objective are summed, $cd(x) = \sum_k cd_k(x)$.

This crowding distance measure is used in crowded tournament selection operator. That is, between two solutions with differing nondomination ranks, we prefer the solution with the lower (better) rank. Otherwise, if both solutions belong to the same front, then we prefer the solution that has higher crowding distance. A solution with a higher value of this distance measure is less crowded by other solutions [7].

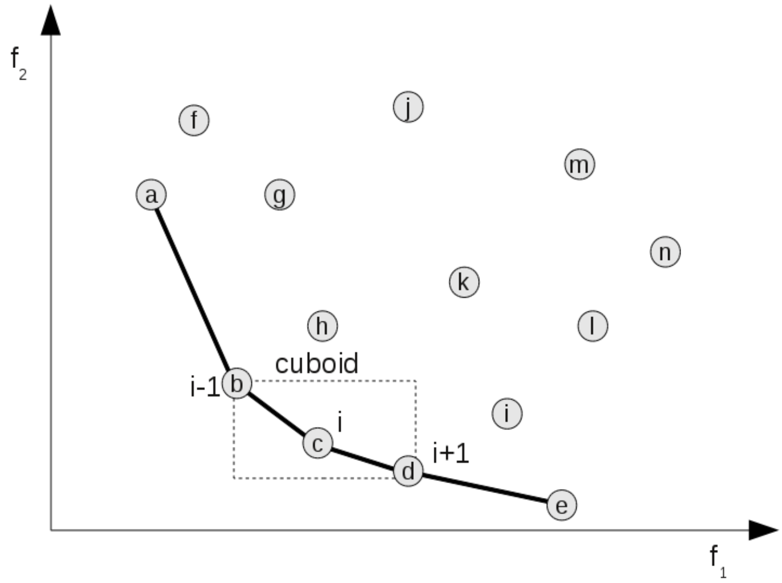

***Fig. 2** Crowding distance calculation [5]*

3 INTEGRATED TIMETABLING AND SCHEDULING IN PUBLIC TRANSPORTATION

In this section, integrated model for timetabling and vehicle scheduling in public transportation is presented. Multiobjective evolutionary algorithm enables that individual objective functions are optimized simultaneously. The basic structure of the model is depicted in Fig. 3.

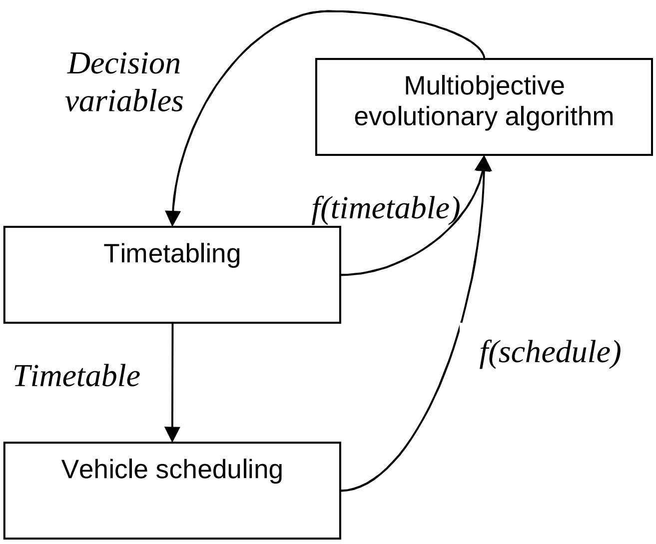

Fig. 3 *Structure of the integrated model*

Decision variables determine the timetables of the lines. The timetable is constructed from decision variables according to the type of timetable. If the timetable is periodical, only the offset of the first departure is needed and the other departures are calculated by adding

multiples of headway to the first departure. In other cases, the decision variables can be exact departure times or offsets from initial timetable. Constructed timetable serves as an input to the vehicle scheduling. The vehicle scheduling can be done by some scheduling model such as assignment model, set partitioning model, time-space networks or by heuristics. In the same time, timetables and vehicle schedules are evaluated and the values of the objective functions are passed over to the evolutionary algorithm. The evolutionary algorithm calculates the dominance ranks and crowding distances and then iteratively repeats the cycle in order to find Pareto optimal values of the decision variables.
The objective function of the timetabling can consist of:

- transfer time of the passengers,
- waiting time on the stops,
- other measures such as evenness of the headway,
- their combination.

The vehicle schedules can be evaluated in terms of:

- number of vehicles needed,
- length of deadhead trips,
- waiting time,
- their combination.

3.1 Test case with one transfer node

Simple example with 4 lines and periodic timetable presented in [8] has two objective functions. The first objective function f_1 returns the tranfer time of passengers:

Minimize $$f_1 = \sum_i \sum_j \sum_k \sum_l \alpha_{ij}(t_{ik} - t_{jl})c_{kl}^{ij}$$

Subject to:

$$(t_{ik} - t_{jl}) \geq 1 \ \ \forall\, i, j, k, l, \alpha_{ij} = 1 \quad (1)$$

$$\alpha_{ij} \in \langle 0,1 \rangle \ \ \forall\, i, j \quad (2)$$

$$t_{ik}, t_{jl} \in Z^+ \ \ \forall\, i, j, k, l \quad (3)$$

$$t_{ik}, t_{jl} \in \langle 0, 60) \ \ \forall\, i, j, k, l \quad (4)$$

Where the transfer time is a difference between the arrival of *k*-th bus/tram of *i*-th route and the arrival of *l*-th bus/tram of *j*-th route, multiplied by number of transfering passengers c_{kl}^{ij} and summed over all buses/trams of all routes . The term α_{ij} can be either 0 or 1 according to the possibility of transfer between routes *i* and *j*. Constraint (1) ensures that transfer time is greater or equal than 1 minute, constraint (2) states that α_{ij} can take binary values and constraint (3) ensures that the arrivals have integer values. The constraint (4) ensures that the arrivals are within the time period of 60 minutes.
The second objective function f_2 evaluates the vehicle schedule in terms of number of vehicles needed to operate the given timetable. Since no interlining is allowed for the vehicles, number of vehicles N to serve one line with headway h_i can be calculated by following formula:

$$N = \frac{l_{AB} + l_{BA} + r_{i,i+1}^A + r_{i,i+1}^B}{h_i}$$

Where l_{AB}, l_{BA} are travel times between terminal stops A and B, $r_{i,i+1}^A$, $r_{i,i+1}^B$ is the difference between the arrival t^A_i, resp. t^B_{i+1} and departure t^A_{i+1}, resp. t^B_i from the terminal.

The algorithm found two solutions in terms of objective functions. The Pareto front of only two points is shown in Fig. 5. The first point has transfer time f_1 = 60 minutes and number of vehicles is f_2 = 21. The second point has worse transfer time f_1 = 64 minutes but lower number of vehicles f_2 = 20.

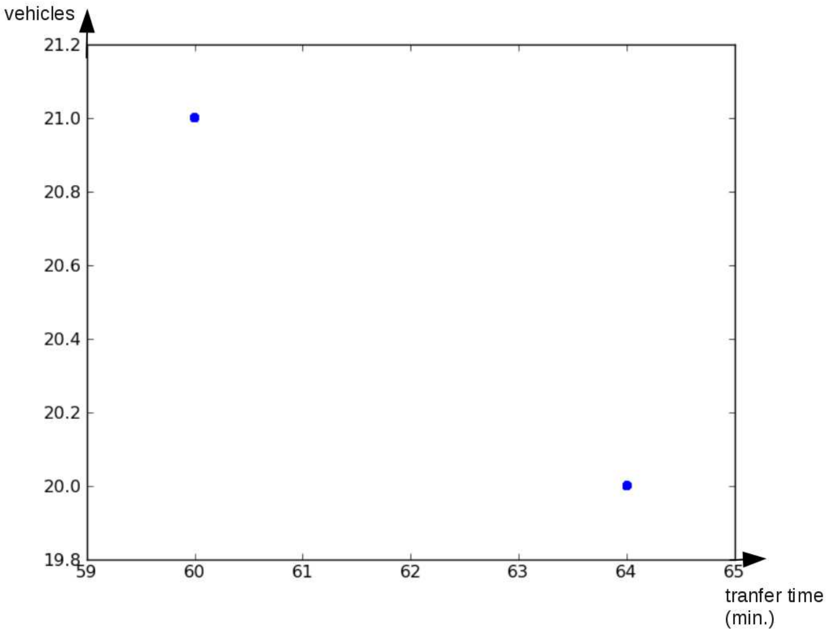

Fig. 4 *Pareto front for test case*

4 FURTHER EXTENSIONS IN FREIGHT TRANSPORTATION

Integrated approach presented above can be applied also in freight transportation. Timetabling and vehicle scheduling in public transportation have parallels in freight transport planning. Especially, this is true when vehicles operate in lines. One example that can illustrate this methodology, is transportation of ready mixed concrete from concrete batch plant to the construction site. Usually, total volume of concrete which has to be transported in certain time period is known in advance. Since the capacity of the vehicles is much smaller than the total volume of the concrete, several deliveries are needed. Depending on the route length and total concrete volume, vehicles can make several deliveries. This transportation resembles one line with certain headway. Other factors such as variable travel times due to the traffic conditions or variable unloading time at the construction site can complicate the process. The key to the concrete transportation problem is the timetable. There are several objectives that should be satisfied:

- minimization of used vehicles,
- minimization of total duration of process,
- minimization of waiting times of vehicles in the construction site, since no or limited simultaneous unloading is allowed,
- minimization of idle time of construction staff due to the waiting for the concrete arrival.

Similarly as in public transport, there are conflicting objectives of the transportation provider and the customer. Therefore, simultaneous optimization of objectives is desirable. The use of Pareto based optimization can help the transportation planner to better explore the alternatives available and to choose the suitable trade-off choice.

5 CONSLUSION

Optimization techniques for solving large and computationally hard problems like evolutionary algorithms enable integrate previously separate stages of the transportation planning process. Furthermore, employing multiobjective approach allows to take multiple objectives into account. This way the conflicting objectives of different stakeholders such as passengers or customers and transportation provider can be optimized simultaneously. Exploring Pareto optimal set helps the decision maker to investigate various scenarios in comparison with traditional methods such as weighted sum method.

Further work will be concentrated on applying this method to real world problems and employing advanced vehicle scheduling techniques such as time-space networks or heuristics.

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