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LOCATING DEPOTS FOR EMPTY CONTAINERS AS A PROBABILISTIC LOCATION ROUTING PROBLEM WITH BALANCING DEMAND

LOCIRANJE DEPOA ZA PRAZNE KONTEJNERE KAO LOKACIJSKI RUTING PROBLEM SA BALANSNIM ZAHTEVIMA

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Abstract: *In this paper, effort in further highlighting the empty logistics units depots location problems has been made. Location problem with balancing demand, analyzed here is considered as probabilistic location routing problem PLRP, formulated and solved as an a priori optimization problem. Due to its complexity exact solution of the problem can be very limited, and because of that solving procedure based on Genetic algorithm has been proposed.*

Key words: *reverse logistics, empty flows, pallets and containers, location-allocation, a priori probabilistic location routing, genetic algorithm*

Apstrakt: *U ovom radu učinjeni su dalji napori na osvetljavanju lociranja depoa za prazne logističke jedinice. Lokacijski problemi sa balansnim zahtevima analizirani u ovom radu, tretirani su kao probabilističi ruting lokacijski problemi, formulisani i rešavani kao a priori optimizacijski problemi. Saglasno kompleksnosti, egzaktno rešavanje ovih problema je ograničeno na probleme manjih dimenzija, pa je u radu predložena procedura bazirana na primeni genetskih algoritama.*

ključne reči: *povratna logistika, tokovi praznih jedinica, palete i kontejneri, lokacija-alokacija, a priori lokacijski ruting, genetski algoritmi*

1 INTRODUCTION

Reverse (return) logistics is usually defined as a system of activities concerned with the care for products and packaging material after they have been used, but in case of reusable packages (which is standard topic of reverse logistics), both, before and after their use. Although processes related to empty reusable logistics units are usually considered as a return logistics activities, those supply and collecting flows are in literature also denoted as "empty units distribution planning". Wu and Dunn (1995) have been defined reverse

1 UVOD

Povratna logistika obično se definiše kao sistem aktivnosti koje se odnose na brigu o proizvodima i ambalaži, nakon njihovog korišćenja, odnosno, kada je reč o ambalaži, i pre i nakon korišćenja. Iako se procesi koji se odnose na prazne logističke jedinice za višekratnu upotrebu obično posmatraju kao aktivnosti povratne logistike, tokovi isporuke i prikupljanja ovih jedinica označavaju se u literaturi i kao «planiranje distribucije praznih jedinica». Wu i Dunn (1995) definišu povratnu logistiku kao otpremu otpadnog materijala od pakovanja, reciklažnog materijala za pakovanje i proizvoda vraćenih od strane korisnika. Slično tome, Fleischmann i drugi (1997),

logistics as shipment of packaging waste, recyclable packages and customers' returns in logistics system. Similarly, Fleischmann at al (1997), in very useful review of quantitative models for reverse logistics, use term "reverse distribution" for process of collection and transportation of used products and packages.

Consequence of the use of returnable units is necessity of activation complex processes that comprise very wide area of different tasks. In the other hand, this requires adequate management decisions and operations. This include: pooling concept considerations; choosing locations for warehousing, repair, recycle and disposal plants; collecting, testing, sorting and transportation strategies considerations; solving allocation and balancing problems with empty units; vehicle routing; solving inventory control problems, etc.

Problem analyzed here is to locate empty containers' depot in order to collect the supply of empty containers available at customers' sites, and to satisfy demand for empty containers. Both should be done under minimal costs. Functions of depots are to provide customer with empty units, to pick up empty units from customers afterwards, and to keep current inventories of containers. Intention here is mostly directed to small containers and pallets, which means that relatively large number of units may be transported on the same vehicle.

In case of pallets and small containers, it is reasonable to consider the empty units depots' location problem as a location routing problem. Namely, in real systems pallet and/or container pools serve different types of customers, from the point of view of demand intensity. Some customers who may be characterized as "small" have low intensity of demand (less-than-truck-load LTL), which means that they may be served by one vehicle, in one multi-stop route. Other customers may be characterized as "large", in sense that their demand has higher intensity (full-truck-load TL), hence they can be served only on straight-and-back basis. Moreover, it is logical to assume that in certain area served by one depot usually exist numerous customers that don't have requirements every day, or in every given interval. From there, in each interval only random set of those customers has requirements, and should be served from the depot. Of course, similar situation is in case of customers denoted here as "large", but because of TL service concept, this does not have impact on depots location.

u veoma dobrom pregledu kvantitativnih modela povratne logistike, za procese sakupljanja i transporta upotrebljenih proizvoda i ambalaže koriste termin "povratna distribucija".

Posledica korišćenja povratnih jedinica jeste neophodnost aktiviranja kompleksnih procesa koji obuhvataju širok spektar različitih zadataka. Sa druge strane, to zahteva adekvatne upravljačke odluke i odgovarajuću realizaciju koje uključuju: razmatranje koncepta pulova; izbor lokacije skladišta, servis za opravku i reciklažu, kao i deponiju za odlaganje otpada; razmatranje strategija prikupljanja, testiranja, sortiranja; rešavanje alokacijskih problema i balansnih zahteva; rutiranje vozila, rešavanje problema upravljanja zalihama i slično.

Problem razmatran u ovom radu odnosi se na lociranje depoa za prazne kontejnere, sa ciljem da se tako obezbedi doprema praznih jedinica od korisnika i da se zadovolji tražnja za praznim kontejnerima i jedno i drugo uz minimalne troškove. Pri tome, funkcija depoa jeste da obezbedi korisnika sa praznim jedinicama, da obezbedi prikupljanje tih jedinica nakon upotrebe, kao i obezbedjenje uskladištenja tekućeg nivoa zaliha kontejnera. Ovaj rad usmeren je prevashodno na razmatranje problema vezanih za male kontejnere i palete, što, pre svega, znači mogućnost da veći broj ovih jedinica bude istovremeno transportovan jednim vozilom.

U slučaju paleta i malih kontejnera problem lociranja depoa za smeštaj ovih jedinica opravdano je tretirati kao lokacijski ruting problem. Naime, u realnim sistemima, paletni i/ili kontejnerski pulovi opslužuju različite tipove korisnika sa aspekta intenziteta tražnje. Odredjeni korisnici koji mogu biti klasifikovani kao "mali" imaju nizak nivo zahteva (manje od jednog vozila LTL[1]), što znači da mogu biti opsluživani istim vozilom, tokom jedne rute sa više zaustavljanja vozila. Drugi deo korisnika mogu se, pak, okarakterisati kao "veliki", u smislu većeg intenziteta zahteva (puno vozilo, TL), pa tako mogu biti opsluživani direktnim isporukama. Šta više, logično je pretpostaviti da u nekom regionu, koji se opslužuje iz jednog depoa, obično postoje korisnici koji ne zahtevaju opslugu svaki dan, odnosno u svakom posmatranom intervalu. U tom slučaju, u svakom od posmatranih intervala biće opsluživan samo deo od ukupnog broja korisnika, koji je slučajnog karaktera. Logično, slična situacija u pogledu redovnosti isporuke, unutar posmatranih intervala, prisutna je i kod "velikih" korisnika, ali zbog tipa opsluge (TL[2]), to nema uticaja na izbor lokacije depoa.

[1] Less than truck load
[2] Truck load

Problem studied here is based on assumption that "small" customers are only "supply customers", which means that empty containers should be picked up from "small customers' sites", and that "large" customers are only "demand customers". That is, in case of large customers, demand for empty containers should be satisfied by delivering required number of empty units. This assumption simplified proposed solving approach, but without significant loss of generality. Moreover, it may be said that customers always can be divided into sets of "mostly demanding", and "mostly supplying" nodes.

In this paper, effort in further highlighting the empty logistics units depots location problems has been made. Location problem with balancing demand, analyzed here is considered as probabilistic location routing problem PLRP, formulated and solved as an a priori optimization problem. In the same time, as "large" customers' nodes are served only on straight-and-back basis, it should be noted that problem actually represents combined location problem with balancing demand, and PLRP with balancing demand. Each of mentioned sub problems is known as NP-hard (facility location problem, vehicle routing problem, a priori probabilistic location routing problem), hence their combination also belongs to the same class. Due to its complexity exact solution of the problem can be very limited, and because of that solving procedure based on Genetic algorithm has been proposed.

Therefore, two main differences from the previous research in this area may be pointed out. The first is that location problem studied here is PLRP with balancing demand, and with no homogeneous customers, where some of them are served on straight-and-back basis and others in the same multi-stop route, which is realized on random set of customers. The second difference from the previous works is in solving approach, since Genetic algorithm based procedure is proposed here.

Remaining of the paper is organized as follows. In Section 2 problem is described in more details, and related literature is summarized. Mathematical formulation is given in Section 3. Solving procedure based on genetic algorithm is proposed in Section 4. Then, the experimental design for computational analysis, together with obtained results is described in Section 5. Finally, conclusions and suggestions for future work are given in Section 6.

Problem razmatran u ovom radu zasniva se na pretpostavci da su "mali" korisnici samo mesta otpreme, što znači da prazne kontejnere treba prikupljati sa mesta gde su "mali" korisnici locirani, a da "veliki" korisnici imaju samo zahteve za prijemom praznih kontejnera, to jest, u slučaju "velikih" korisnika, zahtevi za praznim kontejnerima zadovoljavaju se isporukom zahtevanog broja praznih jedinica. Ova pretpostavka, bez gubitka opštosti, značajno pojednostavljuje predloženi postupak rešavanja problema. Šta više, korisnike je uvek moguće podeliti na skupove "uglavnom isporučioci", odnosno "uglavnom primaoci".

U ovom radu učinjeni su dalji napori na osvetljavanju lociranja depoa za prazne logističke jedinice. Lokacijski problemi sa balansnim zahtevima analizirani u ovom radu, tretirani su kao probabilističi ruting lokacijski problemi, formulisani i rešavani kao a priori optimizacijski problemi. Kako se, istovremeno, "veliki" korisnici snabdevaju po principu direktne isporuke, treba istaći da je, u suštini, reč o kombinovanom lokacijskom problemu sa balansnim zahtevima i probabilističkom ruting lokacijskom problemu. I jedan i drugi od navedenih problema poznati su kao NP-hard (lokacijski problem, problem rutiranja, a priori probabilistički ruting lokacijski problem), pa i njihova kombinacija pripada istoj klasi. Saglasno kompleksnosti, egzaktno rešavanje ovih problema je ograničeno na probleme manjih dimenzija, pa je u radu predložena procedura zasnovana na primeni genetskih algoritama.

Otuda je moguće istaći sledeće dve razlike u odnosu na prethodna istraživanja u ovoj oblasti. Prva je da se u radu razmatra PLRP sa balansnim zahtevima i nehomogenim korisnicima, od kojih se jedan deo opslužuje po principu direktne opsluge, a drugi deo unutar iste rute koja se realizaju na slučajnom skupu korisnika. Naredna razlika je u pristupu rešavanju problema, s obzirom na to da je predložena procedura zasnovana na primeni genetskog algoritma (GA).

Preostali deo rada organizovan je na sledeći način. U tački 2 detaljnije je opisan problem koji se rešava i dat je prikaz literature. Matematička formulacija problema data je u tački 3. Procedura za rešavanje problema, zasnovana na primeni genetskih algoritama predložena je u okviru tačke 4. Računski primeri i dobijeni rezultati opisani su u tački 5. Na kraju, zaključna razmatranja i sugestije za dalji rad prezentirani su u tački 6.

2 PROBLEM DESCRIPTION AND LITERATURE REVIEW

The problem studied here can be defined as follows. A set of nodes: potential depots sites, customers, and their expected demands and supply, together with probabilities of having supply for "small" customers are given. Depending on available supply and demand at customers' nodes, both, demand, and supply customers must not be assigned strictly to one depot. Shipment and pickup routes are carried out by vehicles, which are dispatched from the depots. All pickup routes start from depots, detour randomly generated set of "small" customers and ends at the same depot. Deliveries to demanding customers are realized on straight-and-back basis, from the depots to "large" customers. Thus, all flows must be realized through depots. Direct flows between customers are forbidden. There are also fixed costs associated with opening a depot at certain location, as well as variable costs of depots proportional to the flow realized through a depot. The costs of pickup and delivery operations are linear in the total distance traveled. The problem here is to determine the location of depots and the routes of vehicles from depots to "small" customers, to minimize the sum of location and distribution costs.

The main processes and resulted flows in such system are presented in *Figure 1*. The shown network includes two main types of customers:

- "***Large customers***" (LCs), whose average demand is full truckload, where each delivery vehicle serves only one customer on a straight-and-back basis: depot-customer-depot (demand nodes) and
- "***Small customers***" (SCs), whose average requirement is less than truckload, which means that one vehicle may serve more than one customer on its route (supply nodes).

2 OPIS PROBLEMA I PREGLED LITERATURE

Problem razmatran u ovom radu može biti definisan na sledeći način. Neka je dat skup čvorova: potencijalne lokacije depoa, korisnici i neka su poznati očekivani zahtevi za prijem i isporuku po čvorovima, kao i verovatnoće zahteva za otpremom kod "malih" korisnika. U zavisnosti od zahteva u korisničkim čvorovima, njihova realizacija nije striktno vezana za jedan depo. Direktne isporuke i rute kod prikupljanja jedinica realizuju se vozilima koja su upravljana iz depoa. Rute pri prikupljanju kontejnera počinju u depou, obilaze slučajni broj "malih" korisnika i završavaju se u depou. Isporuke kontejnera do "velikih" korisnika, koji primaju kontejnere, realizuju se po principu direktne isporuke iz depoa. Otuda, svi transportni tokovi realizuju se preko depoa, a direktni tokovi izmedju korisnika nisu dozvoljeni. Prisutni su i fiksni troškovi otvaranja depoa na odredjenoj lokaciji, kao i promenljivi troškovi rada depoa proporcionalni intenzitetu tokova kroz depo. Troškovi dostave i prikupljanja su linearna funkcija predjenog puta. Problem se svodi na odredjivanje lokacije depoa i ruta vozila od depoa do "malih" korisnika, na način da se minimizira suma troškova lociranja i distribucije.

Osnovni procesi i transportni tokovi u jednom takvom sistemu prikazani su na slici 1. Prikazana transportna mreža uključuje dva tipa korisničkih čvorova:

- "***veliki korisnici***" (LC), čiji srednji zahtevi iznose celo vozilo i gde se pri svakoj isporuci opslužuje jedan korisnik po principu direktne isporuke (prijemni čvorovi) i
- "***mali korisnici***" (SC), čiji su srednji zahtevi manji od jednog vozila, što podrazumeva mogućnost opsluge više korisnika u jednoj ruti (čvorovi isporučioci).

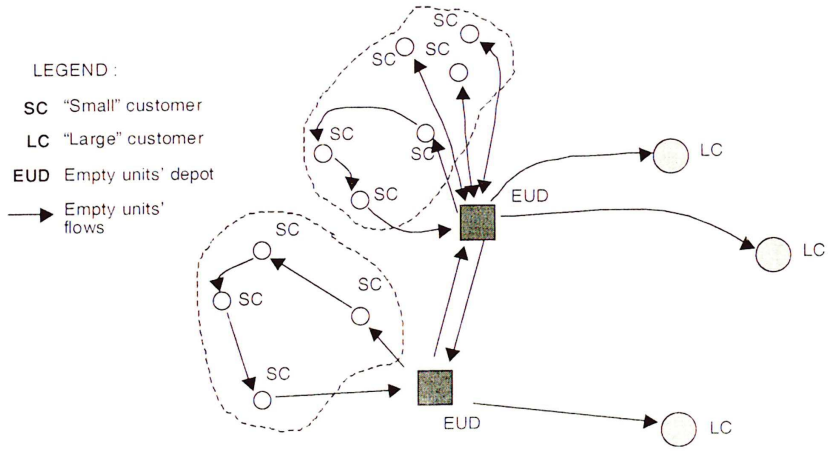

Figure 1 Empty logistic units network
slika 1 Mreža praznih logističkih jedinica

Another specialty is presence of balancing flows between depots. Those balancing flows equate differences between shortages and surpluses of empty logistics units in depots, which may happen occasionally. Functions of depots (EUDs) are to provide "large" customer with empty units, and to pick up empty units from "small" customers afterwards. In that aim, depots should manage the number of available empty units. Depots have two basic "control" instruments for balancing the demand: purchasing the new empty units and/or, repositioning of logistic units from other locations. However, it is assumed that the number of empty logistics units is large enough to satisfy demand in observed period. Also, it is assumed that all logistics units stay in good operative conditions during the observed period. From there, purchasing of the new empty units isn't considered in this paper.

Similar problem, in case of deterministic demand in customer nodes has been formulate by Crainic, Dejax and Delorne (1989), as *"simple multicommodity location-allocation problem with balancing requirements"*. This problem is closely related to well known *"the simple plant location problem"* explained in details in Krarup and Pruzan (1983), with main difference in balancing flows between depots existence. Crainic, Delorne and Dejax (1989) proposed few formulations of the location-allocation problem with balancing requirements, and here is used their idea obtained by fixing binary decision variables which represent depot status. Namely, they shown that mentioned location allocation problem in case of fixed values of depots opening vector becomes incapacitated minimum cost flow problem.

Presence of location routing part of the problem requires additional consideration. Early works in integrated location problems concentrate mostly on the location of single facility. Latter, many attempts were done in solving location routing problems of real size, in case of larger number of customers' nodes and depots. Practical location routing model is described by Madsen (1983), who proposes three heuristics. Srivastava (1993) also suggested other three heuristics. Recently Tuzun and Burke (1999) propose two phase tabu search approach to the location routing problem of real dimensions. Survey of Location routing literature, together with synthesis and classification of the problems in this field may be found in Min, Jayaraman and Srivastava (1998). Generally, although many different approaches exist, natural way of applying metaheuristics like GA, tabu search and simulated annealing is to iterate between

Druga specifičnost jeste postojanje balansnih tokova medju depoima. Balansni tokovi izjednačuju razlike viška i manjka praznih logističkih jedinica koji se povremeno dešavaju u depoima. Funkcija depoa (EUD) je da obezbedjuje "velike" korisnike sa praznim jedinicama i da otprema te jedinice od "malih" korisnika nakon što je prestala potreba za njihovim korišćenjem. U tom cilju u okviru depoa upravlja se brojem raspoloživih jedinica, a to se ostvaruje preko dva "kontrolna instrumenta" koji obezbedjuju izjednačavanje tražnje. Ti instrumenti su nabavka novih praznih jedinica i/ili premestanje jedinica sa drugih lokacija. Medjutim, kako je pretpostavljeno da je broj praznih jedinica dovoljan da zadovolji tražnju u posmatranom periodu, i kako je pretpostavljeno da sve jedinice ostaju u upotrebnom stanju tokom celog perioda posmatranja sistema, nabavka novih jedinica nije uzeta u obzir.

Problem sličan opisanom, u slučaju determinističke tražnje formulisali su Crainic, Dejax i Delorne (1989), kao "standardni lokacijsko – alokacijski problem sa balansnim zahtevima i više artikala". Taj problem tesno je povezan sa "standardnim problemom lociranja postrojenja", koji je detaljno razmotren u radu Krarup and Pruzan (1983), pri čemu se osnovna razlika sastoji u prisustvu balansnih tokova izmedju depoa. Crainic, Dejax i Delorne (1989) preporučuju nekoliko formulacija ovog problema, a ovde je kao osnov poslužila ideja da razmatrani lokacijsko-alokacijski problem, u slučaju poznatih otvorenih lokacija postaje problem minimalnog toka kroz mrežu.

Prisustvo lokacijskog i ruting dela problema zahteva, medjutim, i dodatnu analizu. Rani radovi u oblasti integrisanih lokacijskih problema bili su, uglavnom, koncentrisani na lokaciju jedog objekta. Docnije, činjeni su mnogi pokušaji za rešavanjem lokacijskih ruting problema realnih dimenzija, tj. za slučaj postojanja većeg broja korisnika i depoa. Madsen (1983) opisuje lokacijske i ruting modele koji se mogu koristiti u praktičnoj primeni i predlaže tri heuristike za njihovo rešavanje. Srivastava (1993), takođe, predlaže tri heuristike za rešavanje ovih problema. Nedavno su, Tuzun and Burke (1999) predložili primenu dvofaznog pristupa zasnovanog na primeni "tabu pretraživanja" za rešavanje problema realnih dimenzija. Pregled literature iz oblasti lokacijskih ruting problema, sa sintezom i odgovarajućom klasifikacijom problema može se naći u radu Min, Jayaraman i Srivastava (1998). Generalno gledano, premda postoje različiti pristupi, prirodan put rešavanju ove klase problema primenom GA, tabu pretraživanja i

location and routing phase. Namely, once when location of facilities are fixed, location routing problem reduces to multiple depot vehicle routing problem. However, when on any given instance of the problem only subset of customers' nodes should be visited, as in case of PLRP, there are two possible approaches. One is so called "reoptimization strategy", where tour is optimized for each subset of customers should be visited. Another possible approach is "a priori optimization strategy" where tour is uniquely defined in advance, as a basic sequence of potential visits. As it is mentioned before, a priori solution of probabilistic routing problem is introduced by Jaillet (1985), and studied by Jaillet and Odoni (1988). They modified "nearest neighbor algorithm" and proposed "almost nearest neighbor algorithm" to determine basic sequence of nodes. Also, they analyze "savings algorithm", as well as 2-OPT, and 3-OPT algorithms. Since that time there are several published works directed to a priori optimization strategy in combinatorial optimization problems. Berman and Simchi-Levi (1988) analyze location of Traveling salesman for given node sequence (a priori route), and optimal basic sequence for a given location. They also develop lower bound and branch-and-bound scheme. A comprehensive survey of a priori optimization, together with asymptotic comparison of reoptimization and a priori optimization, theoretical and practical approximations to optimal a priori solutions may be found in paper written by Bertsimas, Jaillet and Odoni (1990). Also, further investigations in this field can be found in Bertsimas and Howell (1993). Laporte, Louveaux and Mercure (1994) formulate a priori Traveling salesman problem as stochastic integer linear program. They reported results for the problems with 10-50 nodes, but in case of "more random" networks (where probabilities of visiting nodes are small), they found that problem tends to be more difficult. Averbakh, Berman and Simchi-Levi (1994) study a priori location routing problem for the five criterions, and they also develop lower bounds. Main results obtained on probabilistic combinatorial optimization problem on graphs are surveyed by Bellalouna, Murat and Paschos (1995).

To the authors' knowledge, there isn't any location-allocation problem which has been defined in such a way when balancing flows between depots exist, and when depots' location is considered as PLRP. It should be noted that problem described here is formulated as single-commodity, not multi-commodity location problem with balancing demand. Approach used here may be applied for multi-commodity problem solving too, but because of simplicity only single-commodity problem has been considered.

simuliranog kaljenja sastoji se u iteraciji izmedju lokacijske i ruting faze. Naime, za fiksirane čvorove sa otvorenim lokacijama, lokacijski ruting problem svodi se na problem rutiranja sa više depoa. Medjutim, u slučaju kada se u svakom intervalu podrazumeva opsluga samo dela korisničkih čvorova, što je slučaj sa PLRP, moguća su dva pristupa. Jedan je poznat kao "reoptimizacija", u kom slučaju se ruta optimizuje za svaki podskup korisnika koji se opslužuje, a drugi se sastoji u primeni "a priori strategije optimizovanja", kada se unapred definiše jedinstvena ruta, kao potencijalni redosled obilaska svih čvorova. A priori strategiju rešavanja probabilističkog ruting problema predložio je Jaillet (1985), a razmatrali su je i Jaillet i Odoni (1988). Ova dva autora modifikovali su algoritam "najbližeg suseda" i predložili algoritam "skoro najbližeg suseda", za odredjivanje polazne sekvence čvorova. Ovi autori analizirali su, takođe, i primenu "algoritma ušteda", kao i 2-OPT i 3-OPT algoritme. Od tada, prisutno je nekoliko radova usmerenih ka primeni a priori strategija optimizacije u kombinatornim problemima. Berman and Simchi-Levi (1988), analiziraju lokaciju trgovačkog putnika za dati redosled obilaska čvorova (a priori ruta), kao i optimalni redosled obilaska čvorova za poznatu lokaciju. Ovi autori utvrdjuju i donju granicu, kao i šemu "grananja i ograničavanja". Obuhvatan pregled a priori optimizacije, uključujući i asimptotsku analizu reoptimizacije i a priori optimizacije, kao i teorijske i praktične aproksimacije a priori rešenja, mogu se naći u radu Bertsimas, Jaillet and Odoni (1990). Dalji rezultati mogu se naći u radu Bertsimas and Howell (1993). Laporte, Louveaux and Mercure (1994) formulišu a priori problem trgovačkog putnika kao problem stohastičkog celobrojnog programiranja. Prezentiraju rezultate za mreže sa 10-50 čvorova, ali za slučaj "više stohastičkih" mreža, kako navode (gde su verovatnoće posete čvora male) uočavaju tendenciju usložnjavanja problema. Averbakh, Berman and Simchi-Levi (1994) studiraju petokriterijumski a priori lokacijski ruting problem i utvrdjuju donju granicu rešenja. Pregled glavnih rezultata u oblasti problema probabilističke kombinatorne optimizacije na grafovima daju Bellalouna, Murat and Paschos (1995).

Prema saznanjima autora ovog rada lokacijsko alokacijski problem opisan ovde kao PLRP sa balansnim zahtevima nije do sada formulisan niti rešavan. Treba takođe naglasiti da formulacija problema koji se razmatra u ovom radu obuhvata jedan artikl (jedan tip praznih logističkih jedinica), premda se izloženi pristup može uz male modifikacije primeniti i na probleme sa više artikala. To međutim, zbog pojednostavljenja ovde nije učinjeno.

3 MATHEMATICAL FORMULATION

Formulation of the PLRP with balancing demand is based on the Crainic, Dejax and Delorne (1989) – "simple multicommodity location-allocation problem with balancing requirements", and LRP formulation used by Tuzua and Burke (1999). Necessary modifications were done accordingly to the type of the problem solved here.

Let be considered set Φ of I nodes which are feasible depots' locations candidates, and set Ω of J customers' nodes. Φ' is the set of opened depots ("active" locations). Let Ξ is the set of all nodes., Obviously, $\Xi = \Phi \cup \Omega$. Further, let Λ denotes the set of "large", and Θ the set of "small" customers' nodes, ($\Omega = \Lambda \cup \Theta$). Also, Λ_n, Λ_m and Θ_m denote, subsets of "large" and "small" customers served from nodes $n, m \in \Phi' \subseteq \Phi$. If transportation costs and costs of establishing and operating depot at every node are known, if average customers' demands characteristics are known, and if distances between nodes are also known, then mentioned problems may be formulated as follows.

Find subset of depots' locations $\Phi' \in \Phi$ which minimize objective functions $Z(\Phi')$

$$Z_L(\Phi') = A + B + C + D, \qquad (1)$$

Where:
A - total costs to transport empty logistics units from depots to "large" customers

$$A = \sum_{i \in \Phi'} \sum_{j \in \Lambda_i} N_{ij} \cdot d_{ij} \cdot C_T , \qquad (2)$$

B - total costs to transport empty units from "small" customers to depots

$$B = \sum_{i \in \Phi'} \bar{d}_i \cdot C_T , \qquad (3)$$

C - total cost of balancing demand realization – cost to transport empty units between depots

$$C = \sum_{i \in \Phi'} \sum_{k \in \Phi'} B_{ik} \cdot d_{ik} \cdot C_T , i \neq k , \qquad (4)$$

D - total cost of establishing and operating a depots

3 MATEMATIČKA FORMULACIJA

Formulacija PLRP sa balansnim zahtevima zasnovana je na Crainic, Dejax i Delornovoj formulaciji "običnog lokacijsko – alokacijskog problema sa balansnim zahtevima i više artikala", i na formulaciji lokacijskog ruting problema prema Tuzua and Burke (1999). Naravno, s obzirom na tip problema koji je ovde rešavan, učinjene su sve neophodne modifikacije.

Neka je Φ skup I čvorova koji predstavljaju dopuštene potencijalne kandidate za lociranje depoa i neka je Ω skup J korisničkih čvorova. Φ' predstavlja skup otvorenih depoa ("aktivnih" lokacija). Neka je Ξ skup svih čvorova. Očigledno, $\Xi = \Phi \cup \Omega$. Dalje, neka je sa Λ označen skup "velikih", i sa Θ skup "malih" korisnika, ($\Omega = \Lambda \cup \Theta$). Takođe, neka Λ_n, Λ_m i Θ_m respektivno označavaju podskupove "velikih" i "malih" korisnika koji se opslužuju iz čvorova $n, m \in \Phi' \subseteq \Phi$. Ako su troškovi transporta, otvaranja i rada depoa u svakom čvoru poznati i ako su poznate karakteristike prosečnih zahteva korisničke tražnje, te ako su poznata rastojanja medju čvorovima na mreži, tada je problem moguće formulisati na sledeći način.

Nalazi se podskup lokacija depoa $\Phi' \in \Phi$ koji minimizuje funkciju cilja $Z(\Phi')$,

$$Z_L(\Phi') = A + B + C + D, \qquad (1)$$

gde su:
A - ukupni troškovi transporta logističkih jedinica od depoa do "velikih" korisnika ,

$$A = \sum_{i \in \Phi'} \sum_{j \in \Lambda_i} N_{ij} \cdot d_{ij} \cdot C_T , \qquad (2)$$

B - ukupni troškovi transporta logističkih jedinica od "malih" korisnika do depoa,

$$B = \sum_{i \in \Phi} \bar{d}_i \cdot C_T , \qquad (3)$$

C - ukupni troškovi realizacije balansnih zahteva troškovi transporta jedinica izmedju depoa

$$C = \sum_{i \in \Phi'} \sum_{k \in \Phi'} B_{ik} \cdot d_{ik} \cdot C_T , i \neq k , \qquad (4)$$

D - troškovi otvaranja i rada depoa

$$D = \sum_{i \in \Phi'} \left[C_F + C_V \cdot \left(\sum_{j \in \Lambda_i} q_{ij} + \sum_{j \in \Theta_i} p_j \cdot q_{ij} \right) \right] . \quad (5)$$

Note that expressions (1), (2), (3), (4), and (5), because of GA as solving algorithm, should have indexes k,n to denote generation (k) and chromosome (n) in k-th generation. However, this was not done to avoid needless ballast in notation. These indexes are used only in diagram presenting algorithm has applied.

This formulation, appropriate for solving by proposed approach, comprise following assumptions and constraints.

(a) Each route starts and ends at the same depot
(b) There are enough empty units in system, so all customers' requirements in observed period may be satisfied without additional supply from outside sources
(c) All empty units stay in good operate conditions through observed period
(d) Total supply flows and total return flows of empty units in the system are approximately equal.
(e) Direct flows between customers are prohibited, and all flows are realized through depots
(f) There are enough vehicles in the system, hence there are no limits in number of available vehicles

Notation:
Φ - $\{i \mid i = 1,\ldots,I\}$ is the set of I nodes, feasible depots' locations candidates
Φ' - set of active depots' location – "opened" depots, $\Phi' \subseteq \Phi$
Ω - $\{j \mid j = 1,\ldots,J\}$ set of J customers' nodes, $\Omega = \Lambda \cup \Theta$
Λ - set of, "large" customers to be served
Θ - set of "small" customers to be served
Ξ - set of all nodes ("large" and "small" customers, and feasible depots' locations), $\Xi = \Phi \cup \Lambda \cup \Theta$
Λ_i - set of "large" customers' nodes served by depot $i \in \Phi'$, $\Lambda_i \subseteq \Lambda$
Θ_i - set of "small" customers' nodes served by depot $i \in \Phi'$, $\Theta_i \subseteq \Theta$
C_T - total transportation costs per trip and per distance unit - vehicle acquiring costs included
C_F - fixed costs of establishing and operating depot (same for all locations)
C_V - variable depot's costs (per unit served by certain depot)

$$D = \sum_{i \in \Phi'} \left[C_F + C_V \cdot \left(\sum_{j \in \Lambda_i} q_{ij} + \sum_{j \in \Theta_i} p_j \cdot q_{ij} \right) \right] . \quad (5)$$

Napomenimo da izrazi (1), (2), (3), (4) i (5), zbog primene GA, zahtevaju dodatno indeksiranje k,n da bi se označila generacija (k) i hromozom (n) u k-toj generaciji. Medjutim, to u okviru formulacije nije učinjeno da bi se izbegao nepotreban balast u označavanju. Navedeni indeksi korišćeni su u okviru prezentiranja algoritma koji je korišćen u procesu rešavanja problema.

Prikazana formulacija, podesna za primenu predloženog pristupa, obuhvata sledeće pretpostavke i ograničenja:

(a) Rute polaze i završavaju se u istom depou.
(b) U sistemu je prisutan dovoljan broj praznih jedinica, tako da je sve zahteve korisnika u posmatranom intervalu moguće realizovati bez nabavke novih jedinica iz spoljnih izvora.
(c) Sve prazne jedinice tokom celog perioda opservacije ostaju u upotrebnom stanju.
(d) Ukupni tokovi prikupljenih jedinica i jedinica koje se otpremaju su približno jednaki tokom posmatranog perioda.
(e) Direktni tokovi praznih jedinica izmedju korisnika nisu dopušteni, već se svi tokovi realizuju preko depoa.
(f) U sistemu je prisutan dovoljan broj vozila, pa ne postoji ograničenje u pogledu raspoloživog broja vozila.

Notacija:
Φ - $\{i \mid i = 1,\ldots,I\}$ skup od I čvorova koji predstavljaju dopuštene lokacije depoa,
Φ' - skup aktivnih lokacija – "otvorenih" depoa, $\Phi' \subseteq \Phi$,
Ω - $\{j \mid j = 1,\ldots,J\}$ skup od J korisničkih čvorova, $\Omega = \Lambda \cup \Theta$,
Λ - skup "velikih" korisnika,
Θ - skup "malih" korisnika,
Ξ - skup svih čvorova na mreži ("veliki" i "mali" korisnici, dopuštene lokacije depoa), $\Xi = \Phi \cup \Lambda \cup \Theta$,
Λ_i - skup "velikih" korisnika koji se opslužuje iz depoa $i \in \Phi'$, $\Lambda_i \subseteq \Lambda$,
Θ_i - skup "malih" korisnika koji se opslužuje iz depoa $i \in \Phi'$, $\Theta_i \subseteq \Theta$,
C_T - ukupni transportni troškovi jedne vožnje na jediničnom rastojanju – uključujući ukupne troškove vozila,
C_F - fiksni troškovi otvaranja i rada depoa (jednaki za sve lokacije),
C_V - varijabilni troškovi rada depoa (po jedinici manipulisane robe),

q_{ij} - demand of customer j served by depot i (expressed in number of units)

q_j - total demand of customer j - $q_j = \sum_{i \in \Phi'} q_{ij}$

N_{ij} - number of trips from the depot i to the customer j, $N_{ij} = \frac{q_{ij}}{Q_V} \cdot p_j$

p_j - probability of offering q_j units for a "small" customer $j \in \Theta$ in observed period (in case of "large" customers, i.e. $p_j = 1, j \in \Lambda$)

b_{ik} - balancing demand between depots i and k (number of empty units being repositioned from depot i to depot k obtained by applying Min. costs flow algorithm

B_{ik} - number of trips from depot i to depot k, related to balancing demand realization

$$B_{ik} = \frac{b_{ik}}{Q_V}, \quad i \neq k$$

d_{ij} - distance between nodes $i, j \in \Xi$

$\overline{d}_i$ - expected length of all routes from the depot i, while serving assigned "small" nodes that belong to subset Θ_i

Q_V - vehicle capacity (expressed in number of empty units)

4 SOLUTION PROCEDURE

In this paper, solution procedure based on genetic algorithms' (GA) has proposed. GA are a class of randomized parallel search heuristics which emulate biological natural selection on a population of feasible solutions. The searching procedure is based on analogy with reproductive and adaptive mechanisms in biological systems. The appliance of genetic mechanisms on optimization of mathematical functions is based on Holland's theoretical works (1975). Later, Goldberg (1989), wrote the most fundamental book about GA and their application. In the last decade, interest for applying GA in many different areas increases permanently, and there are many books and papers in this area. As our intention isn't in giving a comprehensive survey of the related literature, to the interested reader we recommend the most recent book of Gen and Cheng (2000).

GA procedure is applied here as a technique of searching solution space consists of possible combinations of active (opened) depots. For each instance (one combination of open

q_{ij} - zahtevi j-tog korisnika koji se opslužuje iz depoa i (izraženi brojem jedinica),

q_j - ukupni zahtevi j-tog korisnika - $q_j = \sum_{i \in \Phi'} q_{ij}$,

N_{ij} - broj putovanja od i-tog depoa do j-tog korisnika, $N_{ij} = \frac{q_{ij}}{Q_V} \cdot p_j$,

p_j - verovatnoća tražnje q_j jedinica, za "malog" korisnika $j \in \Theta$ u posmatranom intervalu (u slučaju "velikih" korisnika $p_j = 1, \forall j \in \Lambda$),

b_{ik} - balansni zahtevi medju depoima i, k (broj jedinica koje se premeštaju iz depoa i do depoa k utvrdjen primenom Min. costs flow algoritma),

B_{ik} - broj putovanja od depoa i do depoa k, povezan sa realizacijom balansnih zahteva

$$B_{ik} = \frac{b_{ik}}{Q_V}, \quad i \neq k,$$

d_{ij} - rastojanje čvorova $i, j \in \Xi$,

$\overline{d}_i$ - očekivana dužina svih ruta od depoa i, pri opsluzi "malih" čvorova iz podskupa Θ_i,

Q_V - kapacitet vozila (izražen u broju praznih jedinica),

4 PROCEDURA REŠAVANJA PROBLEMA

U radu se predlaže postupak zasnovan na primeni tehnike genetskih algoritama. Genetski algoritmi predstavljaju klasu paralelnih heuristika za slučajno pretraživanje prostora dopuštenih rešenja, zasnovanu na simulaciji biološkog procesa selekcije. Postupak pretraživanja zasnovan je na analogiji sa reproduktivnim i adaptivnim mehanizmima bioloških sistema. Primena genetičkih mehanizama u optimizaciji matematičkih funkcija zasnovana je na Hollands-ovim teorijskim radovima (1975). Docnije, Goldberg (1989) piše fundamentalnu knjigu o GA i njihovoj primeni. U poslednjoj dekadi, interes za primenu GA u različitim oblastima permanentno raste, i prisutan je veliki broj knjiga i radova o tome. Kako ovim radom nije obuhvaćena obuhvatna analiza literature iz ove oblasti, zainteresovanom čitaocu preporučuje se knjiga Gen-a i Cheng-a (2000), koja daje iscrpan prikaz stanja u oblasti, posebno u domenu primene GA u inžinjerstvu..

U ovom radu GA su primenjeni kao tehnika pretraživanja prostora rešenja koji se sastoji od mogućih kombinacija aktivnih (otvorenih depoa). Za svaku analiziranu kombinaciju otvorenih depoa,

depots) value of objective function (1) or (2), then is calculated by solving minimal costs flow problem. A priori route is determined by ANNA-0 algorithm, sub-routes are defined by Routing first – cluster second approach, as in Beasley (1983), but by using expected values. The expected length of routes determination is based on approach proposed by Jaillet and Odoni (1988). Diagram shown in *Figure 2* presents main phases of the solution approach.

The first phase of proposed solving procedure implies randomly generated population of N chromosomes. It means that N combinations of opened locations ("active" depots) should be generated, i.e. N randomly generated subsets $\Phi'_n \subseteq \Phi$, $\Phi'_n \neq \varnothing$, $n=1,2,...,N$. In initial population all subsets are different, i.e. $\Phi'_u \neq \Phi'_v$, $\forall u, v \in \{1,2,...,N\}$. Every subset Φ'_n corresponds to binary coded chromosome of length I, so that each member of chromosome X_i at position i has value 1 only if node $i \in \Phi$ is opened, i.e.

$$X_i = \begin{cases} 1, & \text{if node } i \in \Phi \text{ is opened} \\ 0, & \text{otherwise} \end{cases}, \qquad (6)$$

In the next step, by applying Floyd's algorithm, shortest paths between each pair of source ("small" customers) and destination nodes ("large" customers) are determined. As the Floyd's algorithm determines shortest paths between all pairs of nodes, here was necessary to change distance matrix. All direct distances between customers' nodes are changed to be infinite. In accordance to used notation it means that distances between arbitrary two "small" customers, or two "large" customers is:

$$d_{sp} = \infty, \qquad \forall s, p \in \Omega. \qquad (7)$$

primenom algoritma za rešavanje "minimal costs flow" problem utvrdjuje se vrednost funkcije cilja (1). A priori rute utvrdjuju se primenom ANNA-0 algoritma, a pod-rute primenom Routing first – cluster second pristupa, uz korišćenje očekivanih vrednosti. Očekivana dužina ruta odredjuje se na bazi pristupa predloženog od strane Jaillet and Odoni (1988). Blok dijagram na slici 2 prikazuje glavne faze postupka rešavanja problema.

U prvoj fazi predloženog postupka na slučajan način generiše se populacija od N hromozoma. To znači da je potrebno formirati N kombinacija otvorenih lokacija ("aktivnih depoa"), odnosno podskupova $\Phi'_n \subseteq \Phi$, $\Phi'_n \neq \varnothing$, $n=1,2,...,N$. U početnoj populaciji, svi podskupovi su različiti, tj. $\Phi'_u \neq \Phi'_v$, $\forall u, v \in \{1,2,...,N\}$. Svaki podskup Φ'_n odgovara binarno kodiranom hromozomu dužine I, tako da svaki element hromozoma X_i na mestu i, ima vrednost 1 samo i samo ako je čvor $i \in \Phi$ otvoren ("aktivan"), tj.

$$X_i = \begin{cases} 1, & \text{ako je čvor } i \in \Phi \text{ otvoren} \\ 0, & \text{u suprotnom} \end{cases}, \qquad (6)$$

U narednom koraku, primenom Floyd-ovog algoritma, utvrdjuje se najkraći put izmedju svakog para čvorova. Kako se primenom Floyd-ovog algoritma utvrdjuju najkraći putevi izmedju svih parova čvorova, neophodno je izmeniti matricu rastojanja na način da se rastojanja izmedju korisničkih čvorova prevedu u beskonačna, čime se eliminiše mogućnost direktne veze ovih čvorova. Saglasno korišćenoj notaciji:

$$d_{sp} = \infty, \qquad \forall s, p \in \Omega. \qquad (7)$$

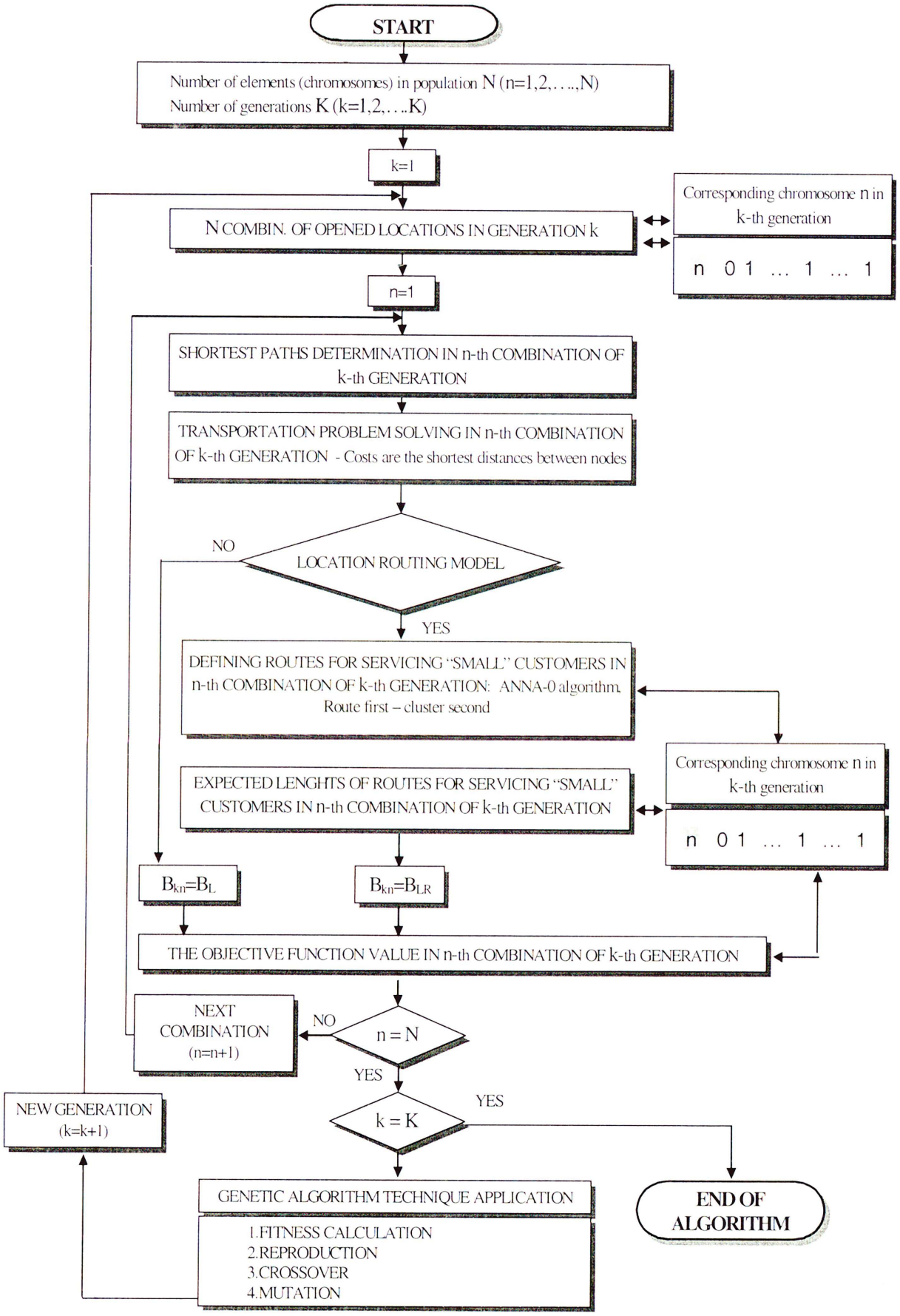

Figure 2 GA approach for solving L& LR problem with balancing demand.
slika 2 GA Algoritam za rešavanje PLRP sa balansnim zahtevima

Among this, distances between all customers and non-opened (inactive) depots were also set to be infinite:

$$d_{cz} = \infty, \quad \forall c \in \Omega, \quad \forall z \in \Phi \setminus \Phi^{I} \ . \ (8)$$

Matter of fact is that all flows in the network, from source to destination must be realized through at least one depot z ($z \in \Phi^{I}$) minimal distances ($d_{xy}^{min}, x \in \Theta, \quad y \in \Lambda$) determined here comprise sequence:

source ("small" customer) – depot 1 – depot 2..... – depot z – destination ("large" customer)

Obtained minimal distances are used than as a transportation costs between a source and destination nodes in transportation problem TLP. In accordance to notation used here, TLP may formulated in following way:

$$\text{Min} \sum_{x \in \Theta} \sum_{y \in \Lambda} d_{xy}^{min} \cdot w_{xy} \qquad (9)$$

subject to standard conditions:

$$\sum_{y \in \Lambda} w_{xy} \le q_x \cdot p_x, \quad \forall x \in \Theta \qquad (10)$$

$$\sum_{x \in \Theta} w_{xy} \le q_y, \quad \forall y \in \Lambda \qquad (11)$$

where w_{xy} is decision variable which denotes quantity of empty units from node x supplies destination node y.

In the solving procedure applied here above constraints were considered as equality conditions because all test problems were adjusted to equal total demand and supply, i.e. $\sum_{x \in \Theta} q_x \cdot p_x = \sum_{y \in \Lambda} q_y$.

After decision variables values are obtained it is possible to determine demand served by currently opened depots. Namely, values of decision variables w_{xy}, together with mentioned shortest paths d_{xy}^{min}, followed by information on all "transfer depots" between source x and destination node y give way of customers' assignment to depots. However, common assumption "each customer is served by one and only one node" here doesn't stand. Any source node may supply arbitrary number of destinations. From there, supplying

Pored toga, veze korisnika i depoa koji nisu aktivni, takođe se ukidaju:

$$d_{cz} = \infty, \quad \forall c \in \Omega, \quad \forall z \in \Phi \setminus \Phi^{I} \ . \ (8)$$

Shodno tome, svi robni tokovi na definisanoj transportnoj mreži moraju biti realizovani najmanje preko jednog depoa. z ($z \in \Phi^{I}$), pa najkraća rastojanja koja se utvrdjuju u okviru algoritma ($d_{xy}^{min}, x \in \Theta, \quad y \in \Lambda$) obuhvataju sekvence:

izvor ("mali" korisnik) – depo 1 – depo 2..... – depo z – odredište ("veliki" korisnik)

Najkraća rastojanja utvrdjena na opisani način koriste se, zatim, kao troškovi transporta izmedju početnih i završnih tačaka toka u procesu rešavanja transportnog zadatka linearnog programiranja TLP, koji se koristi u cilju dodeljivanja korisnika depoima. Saglasno usvojenoj notaciji TLP se može formulisati na sledeći način:

$$\text{Min} \sum_{x \in \Theta} \sum_{y \in \Lambda} d_{xy}^{min} \cdot w_{xy} \ , \qquad (9)$$

respektujući standardna ograničenja:

$$\sum_{y \in \Lambda} w_{xy} \le q_x \cdot p_x, \quad \forall x \in \Theta \ , \qquad (10)$$

$$\sum_{x \in \Theta} w_{xy} \le q_y, \quad \forall y \in \Lambda \ , \qquad (11)$$

gde je w_{xy} promenljiva odlučivanja koja označava količinu (broj) praznih logističkih jedinica kojima se iz čvora x snabdeva čvor y.

U postupku rešavanja opisanog problema prethodna ograničenja (10) i (11) posmatrana su kao uslovi jednakosti, s obzirom na to da su testni problemi formirani na način da se izjednače ukupni zahtevi za prijemom i otpremom jedinica, tj. $\sum_{x \in \Theta} q_x \cdot p_x = \sum_{y \in \Lambda} q_y$.

Nakon što su utvrdjene vrednosti promenljivih odlučivanja, moguće je utvrditi zahteve koji se opslužuju iz trenutno otvorenih depoa. Naime, vrednosti promenljivih w_{xy} zajedno sa prethodno definisanim najkraćim putevima d_{xy}^{min}, koje prati informacija o svim "transfernim" depoima izmedju čvorova x i z, pruža mogućnost za dodeljivanje korisnika depoima. Medjutim, u ovom slučaju ne stoji uobičajena pretpostavka da se "jedan korisnik opslužuje iz samo jednog depoa". Bilo koji izvorišni čvor može snabdevati proizvoljan broj odredišnih,

process may be realized through more than one depot, and two different depots may serve the same customer. Because of that, assignments of the one customer to the more than one depot can be respected by splitting customer nodes into needed number of copies, each with supply (demand) determined by TLP.

As obtained minimal distances comprise sequence of nodes which begins by the nearest depot i_1 to customer x, and ends by the depot i_z nearest to the customer y, becomes clear that flows w_{xy} are also flows between customers' nodes and depots, and between depots themselves.

If sequence $i_1, i_2, \ldots\ldots i_z$ denotes sequence of depots between customers x and y, obviously:

$$w_{xi_1} = w_{i_1 i_2} = \ldots = w_{i_z y} = w_{xy} . \qquad (12)$$

Hence, it is not complicated to calculate total demand served by one depot (needed for obtaining variable depot costs), nor to find demand of small customers are "assigned" to certain depot (needed in routing case). Obviously, demand served by depot is equal to the sum of all flows which go through that depot. Also, summarization of all flows between two depots gives balancing flow between those depots.

In case of location routing part of the model, routes from each depot to all "small" customers assigned to them (in accordance to TLP results) are determined by *"almost nearest neighbor algorithm"*, ANNA-0.

Calculation of expected length of routes is based on idea proposed by Jaillet and Odoni (1988). If depot is indicated by 1, then expected length $\bar{d}_r$ of route r=(1,2,...,n,1) is:

$$\bar{d}_r = \sum_{i=1}^{n}\sum_{j=i+1}^{n} d_{ij} \cdot p_i \cdot p_j \cdot \prod_{k=i+1}^{j-1}(1-p_k) + \sum_{j=1}^{n}\sum_{i=1}^{j-1} d_{ji} \cdot p_i \cdot p_j \cdot \prod_{k=j+1}^{n}(1-p_k) \cdot \prod_{k=1}^{i-1}(1-p_k) . \qquad (13)$$

After that, it is possible to calculate value of objective function. Calculation of total costs to transport empty units from/to all large customers (A) may be done directly by applying equation (2). Total costs to transport empty units from "small" customers (B) can be done by use of expression (3). Also, by applying expressions (4) and (5) it is possible to determine costs of balancing demand (C) and costs of establishing and operating a depots (D).

što znači da se proces snabdevanja može realizovati preko više od jednog depoa, odnosno, dva različita depoa mogu opsluživati istog kòrisnika. Zbog toga, dodeljivanje jednog korisnika većem broju depoa respektovano je fiktivnim uvodjenjem potrebnog broja "kopija" korisnika, pri čemu su intenziteti zahteva za opslugom tih kopija dobijeni rešavanjem TLP.

Kako utvrdjeni najkraći putevi na mreži obuhvataju sekvence koje počinju sa depoom i_1 koji je najbliži korisniku x, a završavaju depoom i_z najbližim korisniku z, jasno je da su tokovi w_{xy} kako tokovi izmedju depoa i korisnika, tako i tokovi izmedju depoa.

Ako sekvenca $i_1, i_2, \ldots\ldots i_z$ označava depoe izmedju korisnika x i z, onda je očigledno da:

$$w_{xi_1} = w_{i_1 i_2} = \ldots = w_{i_z y} = w_{xy} . \qquad (12)$$

Otuda nije teško sračunati ni ukupne zahteve koji se opslužuju iz jednog depoa (što je neophodno za utvrdjivanje varijabilnih troškova rada depoa). Takodje, nije teško utvrditi ni zahteve "malih" korisnika koji su dodeljeni odredjenom depou, što je od značaja za rešavanje ruting dela problema. Očigledno, zahtevi koji se opslužuju iz depoa jednaki su zbiru svih tokova koji prolaze kroz taj depo. Takodje, sumiranjem svih tokova izmedju dva depoa utvrdjuju se balansni zahtevi za posmatrani par čvorova.

Za deo lokacijskog ruting problema, rute od depoa do svakog od "malih" korisnika (alociranih prema odredjenom depou primenom TLP) utvrdjuju se primenom algoritma "skoro najbližeg suseda", ANNA-0.

Proračun očekivane dužine rute zasnovan je na ideji predloženoj od strane Jaillet and Odoni (1988). Ukoliko se depo označi brojem 1, očekivana dužina $\bar{d}_r$ rute r=(1,2,...,n,1) je:

$$\bar{d}_r = \sum_{i=1}^{n}\sum_{j=i+1}^{n} d_{ij} \cdot p_i \cdot p_j \cdot \prod_{k=i+1}^{j-1}(1-p_k) + \sum_{j=1}^{n}\sum_{i=1}^{j-1} d_{ji} \cdot p_i \cdot p_j \cdot \prod_{k=j+1}^{n}(1-p_k) \cdot \prod_{k=1}^{i-1}(1-p_k) . \qquad (13)$$

Nakon toga moguće je sračunati vrednost ciljne funkcije. Proračun ukupnih troškova transporta praznih logističkih jedinica do "velikih" korisnika (A) realizuje se direktno primenom izraza (2). Ukupni troškovi transporta praznih logističkih jedinica do "malih" korisnika (B), realizuju se primenom izraza (3). Slično, primenom izraza (4) i (5) moguće je utvrditi troškove balansnih zahteva (C) i troškove otvaranja i rada depoa (D).

5 NUMERICAL EXPERIMENT

To implement the proposed algorithm non optimized computer program written in Visual Basic has been developed (Figure 3).

5 NUMERIČKI EKSPERIMENT

Za implementaciju predloženog algoritma razvijen je odgovarajući Visual Basic program (slika 3).

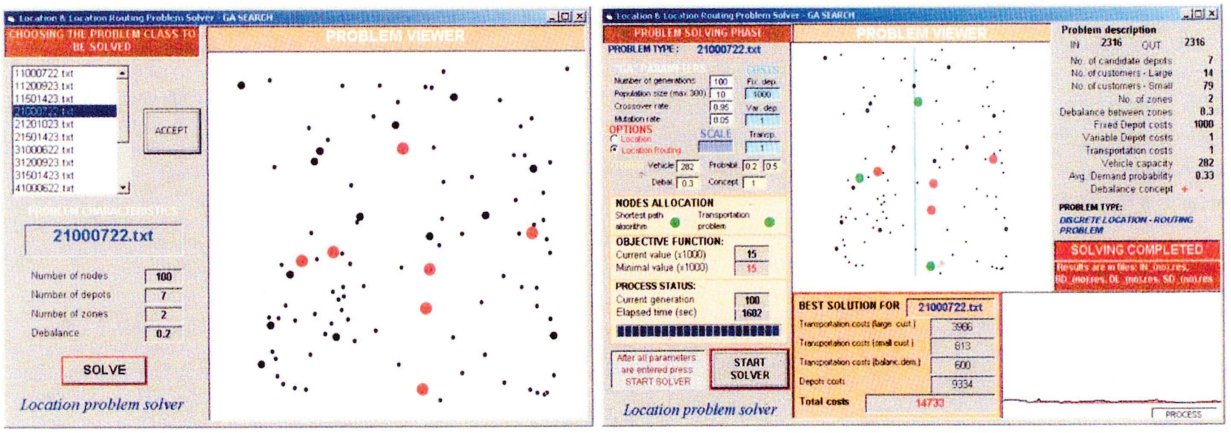

Figure 3 VB software for solving PLRP with balancing demand
slika 3 Izgled VB softvera za rešavanje PLRP sa balansnim zahtevima

With aim to analyze GA solving procedure performances, 120 test problems were generated (60 problems with 100 nodes, and 120 problems with 120 nodes). Problems were solved simultaneously by GA, and enumeration technique. Problems were run on PC Celeron computer 333 MHz. Results have been obtained are shown in table 1. Results show that algorithm proposed, for crossover probability of 0.95, and mutation probability of 0.05, even with small population of only 10 elements, gives very good results, close to optimal solution, after only several generations. Average relative deviation from optimal solution, for all of 120 test problems were 5.1%, for 100 nodes problems 2.1%, and 8.2% for 120 nodes problems. Those results seem to be very good, although they should be accepted with certain reserve, since the test problems were not so large regarding network dimensions.

U cilju analize performansi GA procedure generisano je 120 testnih problema (60 problema sa 100, i 60 sa 120 čvorova). Testni problemi rešavani su paralelno primenom GA algoritma i tehnikom enumeracije. Algoritam je testiran na PC računaru sa Celeron procesorom na 333MHz. Dobijeni rezultati, rezimirani u tabeli 1 pokazuju da predloženi algoritam, za slučaj verovatnoće ukrštanja 0.95 i verovatnoće mutacije 0.05 sa samo deset članova populacije daje veoma dobre rezultate, nalazeći rešenje blisko optimalnom, već posle samo nekoliko iteracija. Prosečno odstupanje rešenja dobijenog primenom GA, u odnosu na optimalno, prema rezultatima analize pomenutih 120 testnih problema iznosi 5.1%, kod problema sa 100 čvorova prosečno odstupanje je 2.1%, a u slučaju mreže sa 120 čvorova to odstupanje je 8.2%, što svakako predstavlja veoma dobar rezultat koji ipak treba uzeti sa rezervom s obzirom na ne tako velike dimenzije testnih problema.

Having in mind all mentioned above, it may be assumed similar algorithm behavior in case of problem instances of larger dimensions. However, larger problems were not solved here particularly because of still no optimized program code, which could impose long running times. Anyhow, mentioned set of problems show that algorithm proposed may be used to solve that class of location problems, having in mind fact that computational time for that class of problems, strategic in their nature, even isn't so important.

S obzirom na prethodno, može se pretpostaviti slično ponašanje algoritma i za probleme većih dimenzija, koji, međutim, ovde nisu rešavani, s obzirom na relativno dugo vreme rada algoritma koje je posledica u najvećoj meri neoptimizovanog programskog koda. Međutim, i analizirani problemi relativno malih dimenzija pokazuju da je razvijeni algoritam moguće primeniti za rešavanje ove klase problema, pri čemu treba imati u vidu i činjenicu da, s obzirom na to da se radi o zadacima strateškog odlučivanja računarsko vreme ovde i nije od presudnog značaja.

Problem generation procedure was based on network where nodes are randomly distributed in square 1000 x 1000. System as whole was

Generator testnih problema bio je baziran na mreži čvorova formiranoj u kvadratu 1000x1000. Sistem kao celina bio je "balansiran" u pogledu prijemno

balanced, and the probabilities of having supply for small customers are determined in accordance to Uniform distribution. Total supply and demand in the network are determined for two zones case, although problem generation software have been developed, offers possibility of different numbers of zones. Supply and demand in nodes are adjusted to certain imbalance level between total supply and demand. Distances between nodes are Euclidean.

otpremnih tokova, a verovatnoće pojave zahteva kod "malih" korisnika generisane su korišćenjem ravnomerne raspodele. Rastojanja čvorova tretirana su kao euklidska, a testni problemi analizirani u ovom radu bili su zasnovani na konceptu dvozonskog sistema, gde izmedju zona postoji odgovarajući debalans. Treba, medjutim, napomenuti da program za generisanje testnih problema izbor broja zona sadrži kao opciju, pa je moguće formirati sistem sa više od dve zone.

Table 1 Performances of proposed GA for solving PLRP with balancing demand on six problems randomly chosen

Tabela 1 Performanse predloženog GA za rešavanje PLRP na primeru nekoliko slučajno izabranih problema

Problem instance	Number of nodes	Number of depot locations	Objective function determined by enumeration	GA performances			
				Objective function determined by GA	Deviation from optimal solution (%)	Computat. time for 100 generations (sec)	Generation in which the best solution was obtained
11000722	100	7	16219	16692	0.4	1491	4
21000722	100	7	14311	14733	2.9	1480	29
31000622	100	6	19876	19876	0	1510	17
11200923	120	9	18508	19336	7.7	2680	32
21201023	120	10	22458	24721	10.1	3175	26
31200923	120	9	22937	23744	3.5	3033	47

6 CONCLUDING REMARKS

Location routing problem solving approach applied here, which is typical for empty containers' depots location, accordingly to the results have been obtained obviously encourage future research effort in this field. Something what may be the biggest challenge here, is impact of the proposed concept of depots location defining on the overall efficiency reverse logistics systems. Namely, the main question is functionality of proposed spatial systems' configurations under real conditions, and particularly how important is impact of balancing flows, resulting from stochastic demand. Answer on those questions would obviously give essential information which are important for solving practical problems of empty containers' depots location.

On the other hand there are several theoretical questions need to be solved. First of all this is a optimality question, and problems of determining lower bound and upper bound of the solution. also, several possibilities for further improving and enhancing the algorithm (another met heuristic applying), also exist.

Very interesting, by itself, is the problem of balancing requirements between depots determining, since those flows in addition to zones

6 ZAKLJUČNA RAZMATRANJA

Pristup rešavanju lokacijskih ruting problema, koji su karakteristični za depoe namenjene skladištenju praznih logističkih jedinica u sistemu povratne logistike, prikazan u ovom radu, shodno dobijenim rezultatima, svakako da ohrabruje dalji nastavak istraživanja. Ono što, možda, može predstavljati i najveći izazov u ovom domenu jeste istraživanje uticaja primene ovakvog koncepta lokacije depoa na ukupnu efektivnost sistema povratne logistike. Naime, postavlja se pitanje kako jedan, na ovaj način prostorno definisan sistem funkcioniše u realnim uslovima, Odnosno, u kojoj meri je izražen uticaj balansnih tokova i stohastičkog karaktera zahteva. Jasno je da odgovori na ova pitanje daju ujedno i suštinski važne odgovore na praktična pitanja lociranja depoa za prazne povratne logističke jedinice.

Sa druge strane, prisutan je i niz interesantnih teorijskih pitanja. Pre svega, postavlja se pitanje mogućnosti utvrdjivanja optimalnog rešenja, odnosno iznalaženja donje i gornje granice rešenja. Takodje, prisutan je i niz mogućnosti za dalje poboljšanje predloženog algoritma, kao i za primenu drugih metaheuristika.

Isto tako, veoma interesantan, sam po sebi, je i problem utvrdjivanja balansnih zahteva, koji pored očekivanih intenziteta koji su posledica debalansa

imbalance, include component implies from stochastic nature of demand, which is not covered by the model. Other directions of future research should be directed to empty and loaded flows integration.

u pojedinim regionima, sadrže i komponentu koja rezultira iz stohastičke prirode tražnje, a prezentiranim modelom nije obuhvaćena. Ostali pravci daljih istraživanja, svakako, mogu biti usmereni ka integraciji povratnih tokova praznih jedinica sa tokovima punih kontejnera.

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