

## CHANGE OF MAXIMUM DEFLECTION AND NATURAL FREQUENCIES AT TRACK ROPE OSCILLATION

## PROMENA MAKSIMALNOG UGIBA I SOPSTVENE KRUŽNE FREKVENCE PRI OSCILOVANJU NOSEĆEG UŽETA

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**Abstract:** In this paper quasi-static simulation of a track rope with bi-cableways was performed. Analysis of the track rope vertical movements, observed in the rope plane, was carried out, and the changes in values of the rope deflection due to the change in the number of cable-cars on the span were defined. Analysis was done for the stationary motion of a haul rope, i.e. of cable-cars. Theoretical results obtained coincide with measurements performed on a constructed cableway. Natural frequencies for oscillations of the rope itself were defined analytically and by the method of final elements.

**Key words:** cableway dynamic, track rope, oscillations, natural frequencies

**Apstrakt:** U radu je izvršena kvazistatička simulacija nosećeg užeta kod dvoužetnih žičara. Vršena je analiza vertikalnih pomeranja nosećeg užeta posmatranog u ravni užeta i određivane su promene vrednosti ugiba užeta usled promene broja kabina na rasponu. Analiza je rađena za stacionarno kretanje vučnog užeta tj. kabina. Dobijeni teorijski rezultati se poklapaju sa merenjima izvršenim na izvedenoj žičari. Određene su sopstvene kružne frekvencije za sopstvene oscilacije užeta analitički i metodom konačnih elemenata.

**Ključne reči:** dinamika žičara, noseće uže, oscilacije, sopstvene frekvencije

### 1 INTRODUCTION

In dynamic analysis of the passenger bi-cableway with cable-cars, the starting point is based on the hypothesis that the track rope bears only vertical load, i.e. cable-cars weight, and that haul (drive) rope bears axial load. This hypothesis can be put only for cableways where the difference in height between the poles /towers/ is considerably smaller than the span length.

Dynamic behaviour of the track rope and vertical movements of the cable-cars can be treated separately from the haul rope dynamic. Free oscillations of the track (static) rope which bears vertical load are analysed. The track rope is the

### 1 UVOD

Kod dinamičke analize dvoužetne putničke žičare sa kabinama polazi se od osnovne pretpostavke da noseće uže prima na sebe samo vertikalno opterećenje tj. težinu kabina, a da vučno (pogonsko) uže prima aksijalno opterećenje. Ovo se sme pretpostaviti kod žičara kod kojih je visinska razlika među stubovima znatno manja od dužine raspona.

Dinamičko ponašanje nosećeg užeta i vertikalna pomeranja kabina se mogu tretirati odvojeno od dinamike vučnog užeta. Analiziraju se slobodne oscilacije nosećeg (nepokretnog) užeta koje prima na sebe vertikalno opterećenje. Noseće uže

rope which is tightened at one of its ends by a defined tension force of the counter-weight, and in that case it can be observed as a beam on two supports.

By cable-cars motion along the span, the number of cable-cars on the span is constantly being changed, so the mass also changes along the span. With the change of height there appears the change of tension, i.e. neither mass matrix nor matrix of rigidity for the track rope are constant.

By means of quasi-static analysis of every span, for the cable way constructed in Singapore (Figure 1), we shall define the change of maximum deflection value at each of the spans depending on the number of cable-cars on the span.

je uže koje je zategnuto na jednom svom kraju određenom zateznom silom od kontratega, i u tom slučaju ono se može posmatrati kao greda na dva oslonca.

Kretanjem kabina duž raspona stalno se menja broj kabina na rasponu, pa se masa menja duž raspona. Sa promenom visine imamo i promenu zatezne sile, tj. ni matrica masa, a ni matrica krutosti za noseće uže nisu konstantne.

Kvazistatičkom analizom svakog raspona, za izvedenu žičaru koja postoji u Singapuru (slika 1), odredićemo promenu vrednosti maksimalnog ugiba na svakom od raspona u zavisnosti od broja kabina na rasponu.

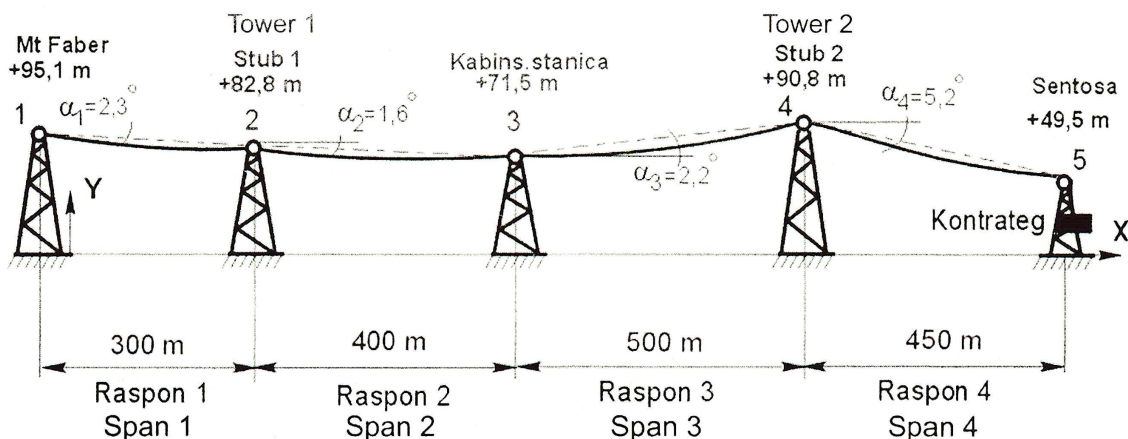


Figure 1 Sketch of the track rope with counterweight on the pole 5  
slika 1 Skica nosećeg užeta sa kontrategom na stubu 5

The cable-cars are distributed along the route at equal intervals  $a$ . At the initial moment of time  $t_0$  the case with minimum number of cable-cars per span is observed, let it be the cable-car  $n$ . The cable-car designated by  $n+1$  is waiting at the support to enter the span and increases the number of cable-cars for  $t > 0$  up to  $n_{max}$  (See Table 1). For fixed  $\Delta t$ , the cable-cars moving is performed and vertical movements of the rope are calculated. Calculated value of the track rope maximum vertical movement i.e. deflection as well as the position of maximum vertical movement along the span (segment  $x$  axes) is obtained for every new position of the cable-car. Simulation is to go on until we obtain again the same situation on the span as it was at moment of  $t_0$ .

Kabine su duž trase raspoređene na podjednaku odstojanju  $a$ . U početnom trenutku  $t_0$  posmatra se slučaj za koji imamo minimalan broj kabina po rasponima, neka je to  $n$  kabina. Kabina označena sa  $n+1$  čeka na osloncu da uđu u raspon i za  $t > 0$  povećava broj kabina do  $n_{max}$  (videti tabelu 1). Za usvojeno  $\Delta t$ , vrši se pomeranje kabina i računaju se vertikalna pomeranja užeta. Sračunata vrednost maksimalnog vertikalnog pomeranja nosećeg užeta odnosno ugib, kao i položaj maksimalnog vertikalnog pomeranja duž raspona (duž  $x$  ose) dobija se za svaki novi položaj kabine. Simulacija se vrši sve dotle dok ponovo ne dobijemo istu sliku na rasponu kao u trenutku  $t_0$ .

**2 CHANGE OF DEFLECTION VALUE  
DEPENDING ON THE CABLE-CAR  
NUMBER ON THE SPAN**

As there is analogy between the curve of the rope deflection and the diagram of bending momentum of the beam on two supports divided by the horizontal component of cable tension, the total moves for sloping spans. (Figure 2) can be defined by the following expressions:

$$y(x) = f_x + xtg(\alpha) = \frac{M_x}{H} + xtg(\alpha),$$

where:

$y(x)$  - vertical move of the rope in the position 'x',

$f_x$  - total rope deflection,

$M_x$  - bending momentum of the beam on two supports given for the position 'x',

$H$  - horizontal component of the tightening force inside the rope.

**2 PROMENA VREDNOSTI UGIBA U  
ZAVISNOSTI OD BROJA KABINA NA  
RASPONU**

Kako postoji analogija između krive ugiba užeta i dijagrama momenta savijanja grede na dva oslonca deljenog sa horizontalnom komponentom zatezne sile, ukupna pomeranja za nagnute raspone (slika 2) mogu se odrediti uz pomoć izraza:

$$y(x) = f_x + xtg(\alpha) = \frac{M_x}{H} + xtg(\alpha),$$

gde je:

$y(x)$  - vertikalno pomeranje užeta u položaju x,

$f_x$  - ukupan ugib užeta,

$M_x$  - moment savijanja grede na dva oslonca napisan za položaj x,

$H$  - horizontalna komponenta zatezne sile u užetu.

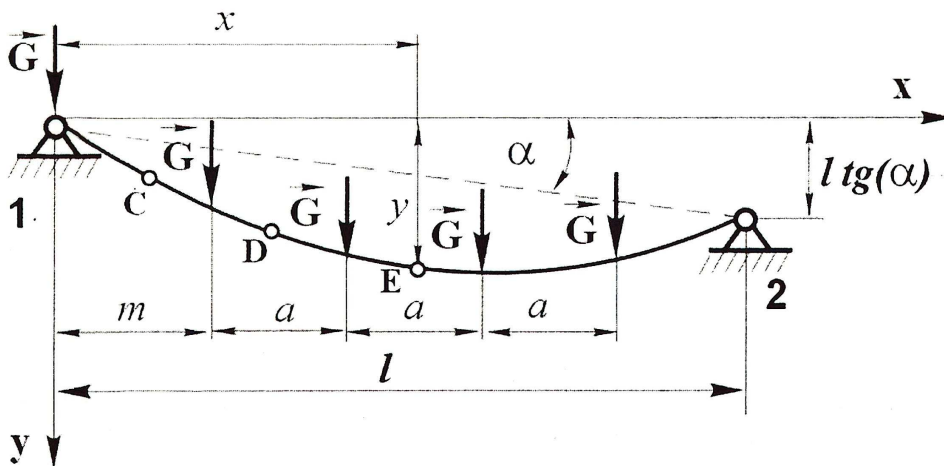


Figure 2 A span of the cableway loaded by the cable-car weight  
slika 2 Jedan od raspona žičare opterećen težinom kabina

Applying the principle of superposition, we obtain for the total deflection:

$$f_x = f'_x + f''_x,$$

$f'_x$  - deflection due to rope weight per length unit,

$f''_x$  - deflection due to concentrated forces, i.e., to the cable-car weight.

Primenom principa superpozicije, dobijamo za ukupan ugib:

$$f_x = f'_x + f''_x,$$

$f'_x$  - ugib od težine užeta po jedinici dužine,

$f''_x$  - ugib od koncentrisanih sila tj. težine kabine.

General formula for the vertical move  $y(x)$ , at whatever point of the span along axis  $x$ , for  $n$  cable-cars on the span which are of the same weight and at the same distance from each other (Figure 2), is obtained by calculating the momentum for the points C, D, E, etc. The repeating terms can be generalised as follows:

$$y(x) = \frac{G}{H} \left[ x(n-u) - m \left( \frac{xn}{l} - u \right) - a \left( \frac{xb}{l} - c \right) \right] + \frac{qx(l-x)}{2H} + xtg(\alpha), \quad (1)$$

where is:

$y(x)$  - vertical move of the rope in the position 'x',

$G$  - cable-car weight,

$H$  - horizontal component of the cable tension inside the rope,

$n$  - the total number of cable-cars on the span,

$$b = \frac{n(n-1)}{2},$$

$u$  - the number of cable-cars to the left from the point on axis 'x' for which deflection is calculated,

$$c = \frac{u(u-1)}{2},$$

$m$  - horizontal distance from the left support to the first cable-car,

$a$  - distance between the cable-cars measured in horizontal direction,

$l$  - distance between the supports measured in horizontal direction,

$q$  - rope weights per length unit (specific weight),

$\alpha$  - angle of incline- between the supports.

From the general expression (1) maximum vertical deflections can be found, i.e. deflections and it is necessary to locate the  $y_{max}$  with respect to first, second, third or  $n$  cable-car. By means of a quasi-static analysis of the model in Figure 2, where for every move of  $\Delta s$ , i.e.  $\Delta t = 0.1s$ , new value of maximum deflection is obtained, as well as its position along the span. By introducing every calculated point, the following diagrams are obtained (analysis was carried out during stationary motion of the cableway at the speed of 2.5 m/s).

Opštu formulu za vertikalno pomeranje  $y(x)$ , u bilo kojoj tački raspona po  $x$  osi, za  $n$  kabina na rasponu koje su jednakih težina i na podjednakom odstojanju jedna od druge (slika 2), dobijamo pisanjem momenata za tačke C, D, E itd. Uočavaju se članovi koji se ponavljaju i izraz se može uopštiti:

gde je:

$y(x)$  - vertikalno pomeranje užeta u položaju  $x$ ,

$G$  - težina kabine,

$H$  - horizontalna komponenta zatezne sile u užetu,

$n$  - ukupan broj kabina na rasponu,

$$b = \frac{n(n-1)}{2},$$

$u$  - broj kabina levo od tačke na  $x$ -osi za koju računamo ugib,

$$c = \frac{u(u-1)}{2},$$

$m$  - horizontalno rastojanje od levog oslonca do prve kabine,

$a$  - odstojanje između kabina mereno u horizontalnom pravcu,

$l$  - rastojanje između oslonaca mereno u horizontalnom pravcu,

$q$  - težine užeta po jedinici dužine (specifična težina),

$\alpha$  - ugao nagiba među osloncima.

Iz opšteg izraza (1) mogu se naći maksimalna vertikalna pomeranja tj. ugibi i treba locirati da li se  $y_{max}$  nalaze između prve, druge, treće ili  $n$ -te kabine. Kvazistatičkom analizom modela sa slike 2, gde za svako pomeranje  $\Delta s$ , odnosno  $\Delta t = 0.1s$ , dobijamo novu vrednost maksimalnog ugiba kao i njegov položaj duž raspona. Unošenjem svake izračunate tačke dobijaju se sledeći dijagrami (analiza je rađena tokom stacionarnog kretanja žičare brzinom od 2,5 m/s).

In Table 1 maximum number of cable-cars for some spans is given.

U Tabeli 1 dat je maksimalan broj kabina po pojedinim rasponima.

Table 1 maximum number of cable-cars

Span	$n_{max}$ - maximum number of cable-cars
1	5
2	7
3	9
4	8

tabela 1 maksimalan broj kabina

Raspon	$n_{max}$ - max. broj kabina po rasponu
1	5
2	7
3	9
4	8

Span 1 (at  $t_0 : n = 4$  do  $n_{max} = 5$  cable-cars on the span, horizontal distance between the supports is 300m)

Raspon 1 (u  $t_0 : n = 4$  do  $n_{max} = 5$  kabina na rasponu, horizontalno odstojanje među osloncima 300m)

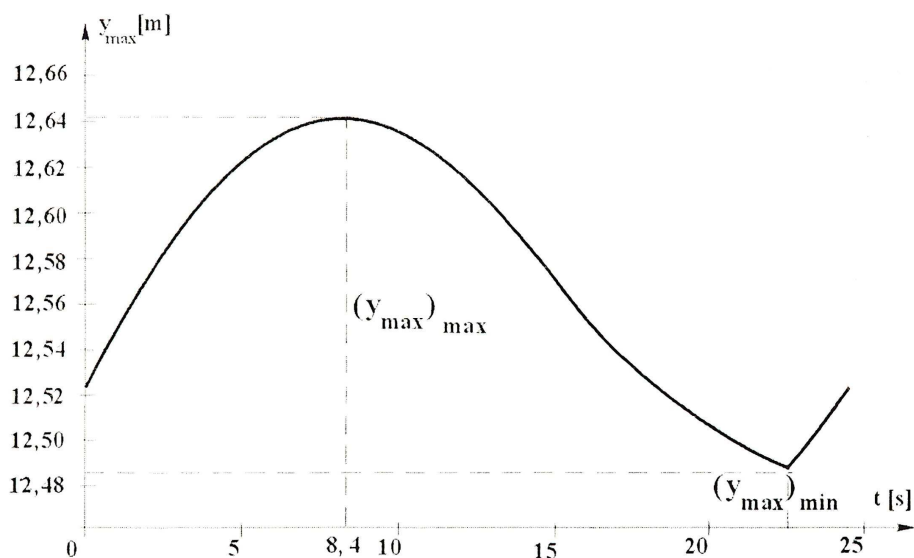


Figure 3 Diagram of the maximum move in the direction  $y$  in the function of time, at span 1  
slika 3 Dijagram maksimalnog pomeranja u  $y$  pravcu u funkciji vremena, na rasponu 1

In Figure 3 one can see that maximum move in the direction  $y$  is about 12.52 m when 4 (four) cable-cars are on the span at the moment of time  $t_0 = 0$  s. On entering the fifth cable-car on the span 1, there is an increase of the coordinate 'y' and maximum move rises to the value of 12.64 m for  $t = 8,6$  s, more precisely, the fifth cable-car has covered the route of  $s = 21,5$  m. After that, there is a decrease of coordinate 'y' right to the moment of time  $t = 22,4$  s when one cable-car leaves span one, shifts to span two where, after all that, there are again four cable-cars on the span.

Sa slike 3 se vidi da je maksimalno pomeranje u  $y$  pravcu kada se na rasponu nalaze 4 kabine oko 12,52 m u trenutku  $t_0 = 0$  s. Ulaskom 5-te kabine na raspon 1, imamo porast  $y$  koordinate i maksimalno pomeranje raste do vrednosti 12,64 m za  $t = 8,6$  s, tačnije 5-ta kabina je prešla put od  $s = 21,5$  m. Nakon toga imamo pad  $y$  koordinate sve do trenutka  $t = 22,4$  s kada jedna kabina napušta raspon jedan, prelazi na raspon dva gde nakon toga opet imamo 4 kabine na rasponu.

Figure 4 shows where the maximum deflection is on axis 'x' at any moment of time. It means that the figure shows the position change of maximum deflection along the span, i.e. along direction 'x' in time.

Slika 4 prikazuje gde se u svakom trenutku po x-osi nalazi maksimalan ugib. Znači, na slici je prikazana promena položaja maksimalnog ugiba duž raspona tj. x pravca u vremenu.

Finally, Figure 5. shows the change of rope deflection in the function of time  $f = f(t)$ . That diagram is obtained by the following expression:

Konačno, slika 5 prikazuje promenu ugiba užeta u funkciji vremena  $f = f(t)$ . Taj dijagram se dobija uz pomoć izraza:

$$f = y_{max} - x \cdot tg \alpha$$

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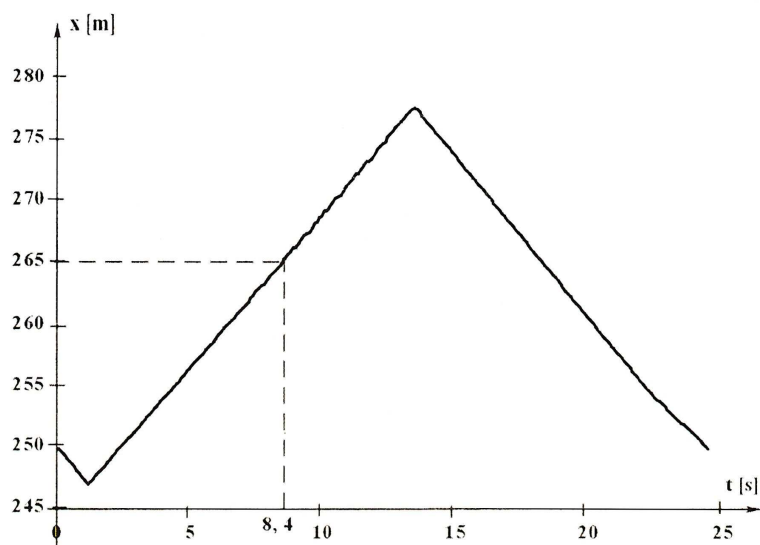


Figure 4 Diagram of the position change of the maximum move  $y_{max}$  along direction  $x$  in the function of time, span 1

slika 4 Dijagram promene položaja maksimalnog pomeranja  $y_{max}$  duž  $x$  pravca u funkciji vremena, raspon 1

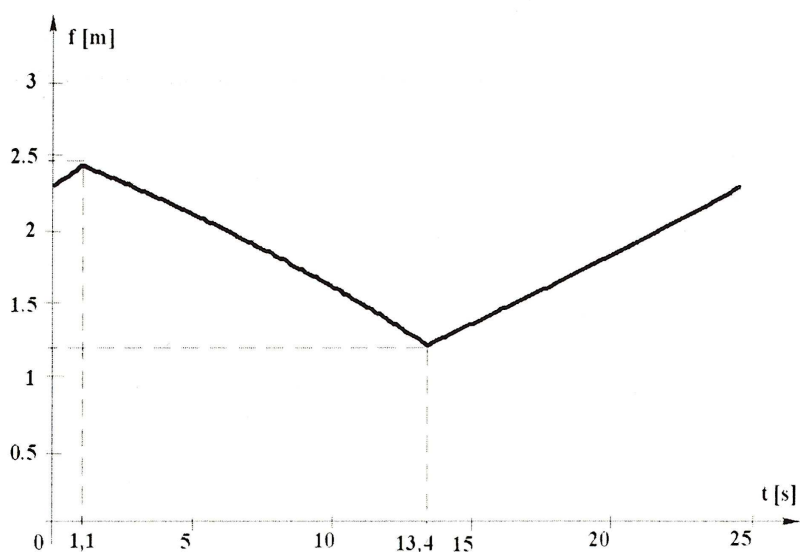


Figure 5 Diagram of the rope deflection as the function of time, span 1  
slika 5 Dijagram promene ugiba kao funkcije vremena, raspon 1

Span 2 (at  $t_0 : n = 6$  until  $n_{max} = 7$  cable-cars on the span, horizontal distance between the supports is 400m)

Raspon 2 (u  $t_0 : n = 6$  do  $n_{max} = 7$  kabina na rasponu, horizontalno odstojanje među osloncima 400m)

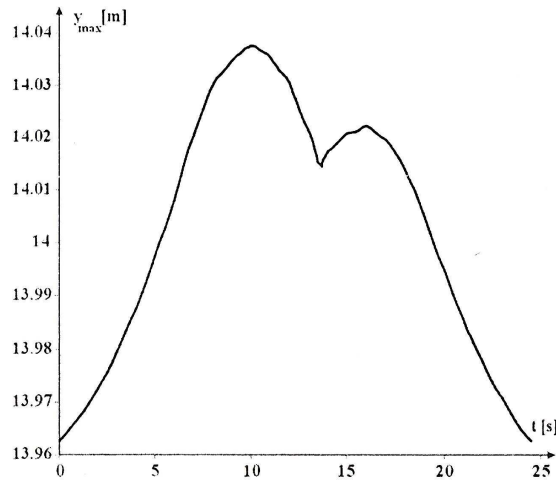


Figure 6 Diagram of the maximum move in the direction  $y$  in the function of time, at span 2  
slika 6 Dijagram maksimalnog pomeranja u  $y$  pravcu u funkciji vremena, na rasponu 2

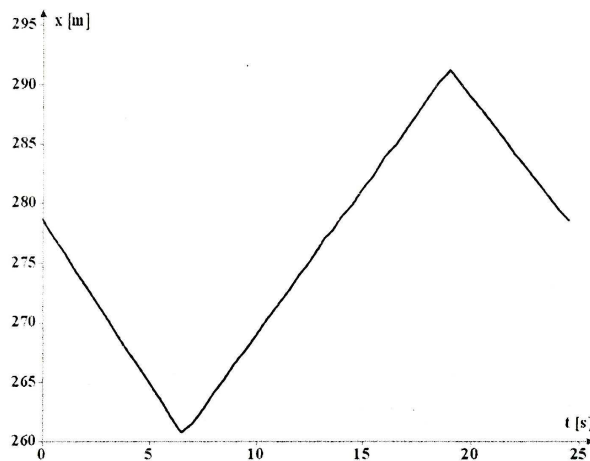


Figure 7 Diagram of the position change of the maximum move  $y_{max}$  along direction  $x$  in the function of time, span 2

slika 7 Dijagram promene položaja maksimalnog pomeranja  $y_{max}$  duž  $x$  pravca u funkciji vremena, raspon 2

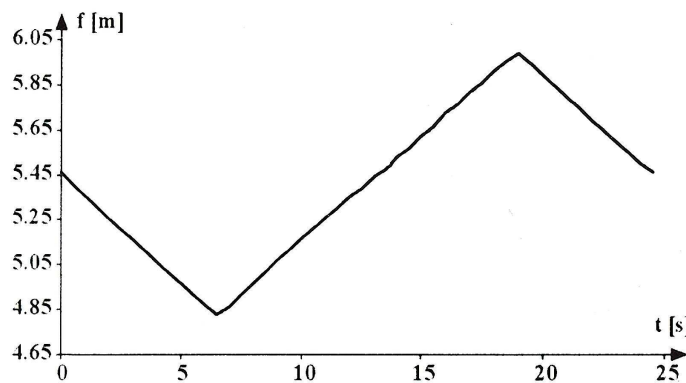


Figure 8 Diagram of the deflection change as the function of time, span 2  
slika 8 Dijagram promene ugiba kao funkcije vremena, raspon 2

- Span 3 (at  $t_0 : n = 8$  to  $n_{max} = 9$  cable-cars on the span, horizontal distance between the supports is 500m) -

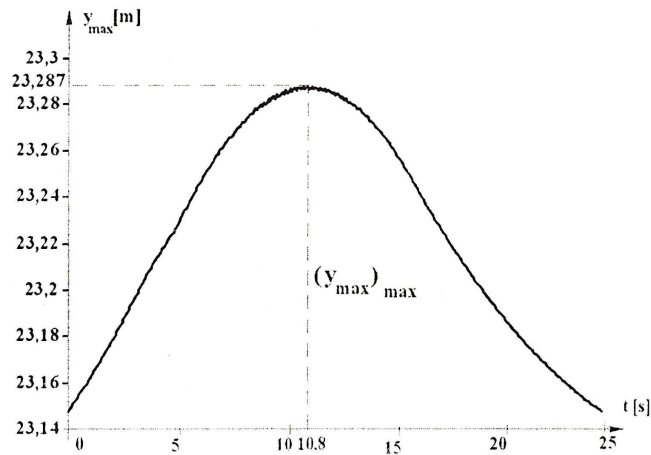


Figure 9 Diagram of the maximum move in the direction  $y$  in the function of time, at span 3  
slika 9 Dijagram maksimalnog pomeranja u  $y$  pravcu u funkciji vremena, na rasponu 3

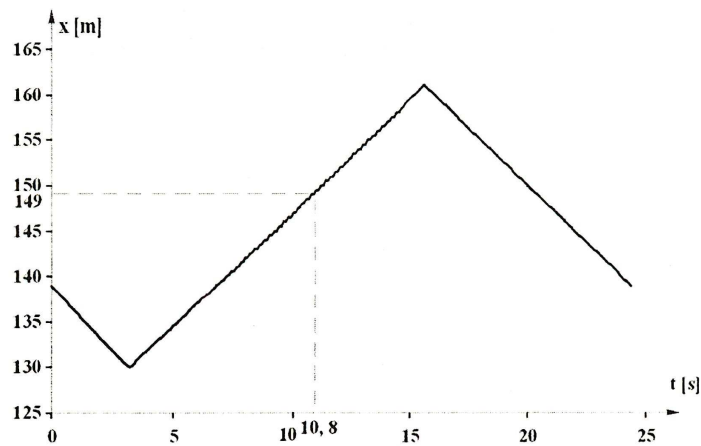


Figure 10 Diagram of the position change of the maximum move  $y_{max}$  along direction  $x$  in the function of time, span 3

slika 10 Dijagram promene položaja maksimalnog pomeranja  $y_{max}$  duž  $x$  pravca u funkciji vremena, raspon 3

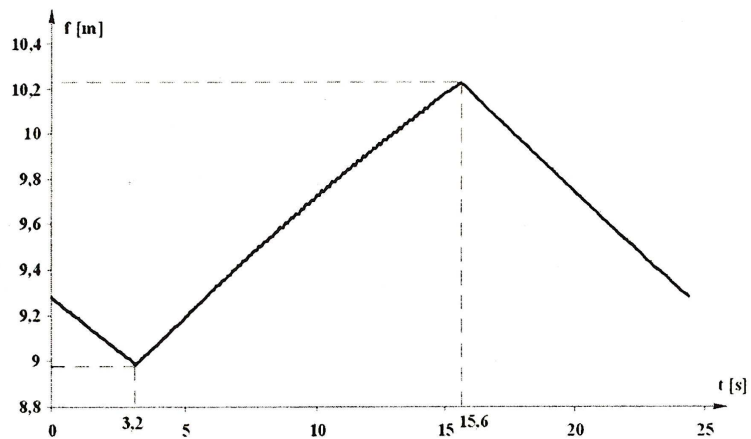


Figure 11 Diagram of the deflection change as a function of time, span 3  
slika 11 Dijagram promene ugiba kao funkcije vremena, raspon 3



- Span 4 (7-8 cable-cars on the span, horizontal distance between the supports is 450m)

The towers are placed in such a way that, regardless of the change in the number of cable-cars on the span 4, change of the maximum on the parabola is outside the span.

These simulated vertical moves (Figures 3, 6, and 9) that appeared as a result of the changeable number of cable-cars on the span, with stationary motion of cable-cars, i.e. of the haul rope, are of the same order of values as the measurements that are in the paper [1].

### 3 DEFINING OF THE NATURAL FREQUENCY OF THE HAUL ROPE FREE OSCILLATIONS

Natural frequencies of free oscillations can be defined in two ways:

- Analytically, and
- By the method of final elements.

According to recommendation of some authors and manufacturers of ropes (USSteel wire rope) when the span includes more than five cable-cars, concentrated masses of cable-cars can be with sufficient preciseness in calculation substituted by the total mass of the rope per length unit:

$$m' = m'_{nu} + \frac{m_k}{l_i},$$

where:

$m'_{nu}$  - track rope mass per length unit,

$m_k$  - cable-car mass,

$l_i$  - distance between two adjacent cable-cars.

#### a) Analytical way of solving the problem:

Expressions have been taken from the paper [2], and they also appear in the papers [1] and [3], and the lowest natural frequency of oscillation for symmetrical mode of oscillation is:

$$f_1 = \frac{b}{2\pi} \sqrt{\frac{H_x}{m'}} \text{ [Hz]},$$

- Raspon 4 (7-8 kabina na rasponu, horizontalno odstojanje među osloncima 450m) -

Stubovi su tako postavljeni da se bez obzira na promenu broja kabina na rasponu 4, promena maksimuma na paraboli nalazi van raspona.

Ova simulirana vertikalna pomeranja (slike 3, 6 i 9) nastala kao posledica promenljivog broja kabina na rasponu, kod stacionarnog kretanja kabina tj. kretanja vučnog užeta, istog su reda veličina kao i merenja koja se nalaze u radu [1].

### 3 ODREĐIVANJE SOPSTVENE KRUŽNE FREKVENCIJE SLOBODNIH OSCILACIJA NOSEĆEG UŽETA

Sopstvene kružne frekvencije slobodnih oscilacija mogu se odrediti na dva načina:

- analitički i
- metodom konačnih elemenata.

Prema preporukama pojedinih autora i proizvođača užadi (USSteel wire rope), kada raspon ima više od 5 kabina, u proračun se sa dovoljnom tačnošću mogu koncentrisane mase od kabina zameniti ukupnom masom užeta po jedinici dužine:

$$m' = m'_{nu} + \frac{m_k}{l_i},$$

gde je:

$m'_{nu}$  - masa nosećeg užeta po jedinici dužine,

$m_k$  - masa kabine,

$l_i$  - rastojanje između dve susedne kabine.

#### a) Analitički način rešavanja:

Izrazi su preuzeti iz rada [2], a pojavljuju se i u radovima [1] i [3], i najniža sopstvena kružna frekvencija oscilovanja za simetrični mod oscilovanja je:

$$f_1 = \frac{b}{2\pi} \sqrt{\frac{H_x}{m'}} \text{ [Hz]},$$

where:  $b$  - the lowest root of the frequency equation is given by expression:

$$\operatorname{tg}\left(\frac{bL_x}{2}\right) = \frac{bL_x}{2} - \frac{4}{\lambda^2} \left(\frac{bL_x}{2}\right)^3,$$

where:

$H_x = H \sec \alpha$  - horizontal component of the cable tension inside the rope, projected onto local axis 'x' (along the chord of the span), Figure 12,

$L_x = L \sec \alpha$  - horizontal distance between two supports projected onto local axis 'x' of the rope (Figure 12.),

$$\lambda^2 = \left(\frac{m'gL \cos \alpha}{H}\right)^2 \frac{EAL}{HL_E} \text{ - non-dimensional}$$

value, takes into account influence of deflection, angle of incline and axial rigidity on dynamic behaviour,

$$L_E = L \left[ 1 + \frac{1}{8} \left(\frac{m'gL \cos \alpha}{H}\right)^2 \right] = L \left[ 1 + 8 \left(\frac{d}{L}\right)^2 \right] \text{ -}$$

statically (extended) rope length,

$$d = \frac{(m'gL^2 \cos \alpha)}{8H} \text{ - maximum deflection.}$$

gde je:  $b$  - najmanji koren frekventne jednačine date izrazom

$$\operatorname{tg}\left(\frac{bL_x}{2}\right) = \frac{bL_x}{2} - \frac{4}{\lambda^2} \left(\frac{bL_x}{2}\right)^3,$$

gde su:

$H_x = H \sec \alpha$  - horizontalna komponenta zatezne sile u užetu, projektovana na lokalnu x-osu (duž tetive raspona), slika 12,

$L_x = L \sec \alpha$  - horizontalno rastojanje između dva oslonca projektovano na lokalnu x-osu užeta (slika 12),

$$\lambda^2 = \left(\frac{m'gL \cos \alpha}{H}\right)^2 \frac{EAL}{HL_E} \text{ - bezdimenziona}$$

veličina, uzima u obzir uticaj ugiba, ugla nagiba i aksijalne krutosti na dinamičko ponašanje,

$$L_E = L \left[ 1 + \frac{1}{8} \left(\frac{m'gL \cos \alpha}{H}\right)^2 \right] = L \left[ 1 + 8 \left(\frac{d}{L}\right)^2 \right] \text{ -}$$

statički (izdužena) dužina užeta,

$$d = \frac{(m'gL^2 \cos \alpha)}{8H} \text{ - maksimalan ugib.}$$

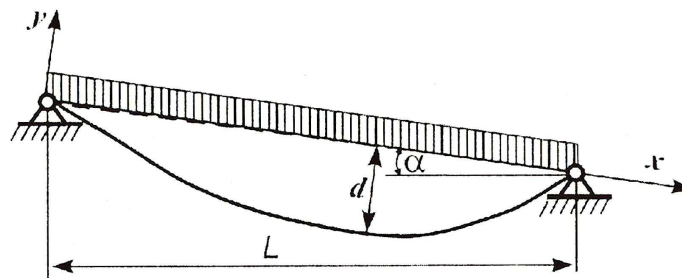


Figure 12 The rope tightened at both ends  
slika 12 Uže zategnuto na dva kraja

The other, anti-symmetrical mode at about the middle of the span or the second natural frequency:

$$f_2 = \frac{1}{L_x} \sqrt{\frac{H_x}{m'}} \text{ [Hz].}$$

Drugi, antisimetrični mod oko sredine raspona ili druga sopstvena kružna frekvencija:

$$f_2 = \frac{1}{L_x} \sqrt{\frac{H_x}{m'}} \text{ [Hz].}$$

**b) Defining natural frequencies by the method of final elements:**

In order to apply the program for defining natural frequencies to find the solution, for the case of the constructed cableway shown in the Figure 1. , it was necessary for every span that is discreted by a certain number of final elements to introduce the matrix of rigidity  $[K_E]$  for each element. Matrix of rigidity  $[K_E]$  given in the paper [1], for the element of the length  $L_E$ , loaded by the tension force  $T$  and by bending rigidity  $I$ , in the local coordinate system, for each element has the following form:

**b) Određivanje sopstvenih frekvencija metodom konačnih elemenata:**

Da bi korišćenjem programa za određivanje sopstvenih kružnih frekvenci došli do rešenja, za slučaj izvedene žičare date na slici 1, bilo je neophodno da se za svaki raspon koji je diskretizovan određenim brojem linijskih konačnih elemenata unese matrica krutosti  $[K_E]$  svakog elementa. Matrica krutosti  $[K_E]$  data u [1], za element dužine  $L_E$ , opterećena zateznom silom  $T$  i savojnom krutošću  $I$ , u lokalnom koordinatnom sistemu, za svaki od elemenata izgleda ovako:

$$[K_E] = \begin{bmatrix} \frac{EA}{L_E} & 0 & 0 & -\frac{EA}{L_E} & 0 & 0 \\ 0 & \frac{12EI}{L_E^3} + \frac{T}{L_E} & \frac{6EI}{L_E^2} & 0 & -\frac{12EI}{L_E^3} - \frac{T}{L_E} & \frac{6EI}{L_E^2} \\ 0 & \frac{6EI}{L_E^2} & \frac{4EI}{L_E} & 0 & -\frac{6EI}{L_E^2} & \frac{2EI}{L_E} \\ -\frac{EA}{L_E} & 0 & 0 & \frac{EA}{L_E} & 0 & 0 \\ 0 & -\frac{12EI}{L_E^3} - \frac{T}{L_E} & -\frac{6EI}{L_E^2} & 0 & \frac{12EI}{L_E^3} + \frac{T}{L_E} & -\frac{6EI}{L_E^2} \\ 0 & \frac{6EI}{L_E^2} & \frac{2EI}{L_E} & 0 & -\frac{6EI}{L_E^2} & \frac{4EI}{L_E} \end{bmatrix}$$

Main forms of the track rope oscillation are obtained:  
Dobijaju se glavni oblici oscilovanja nosećeg užeta:

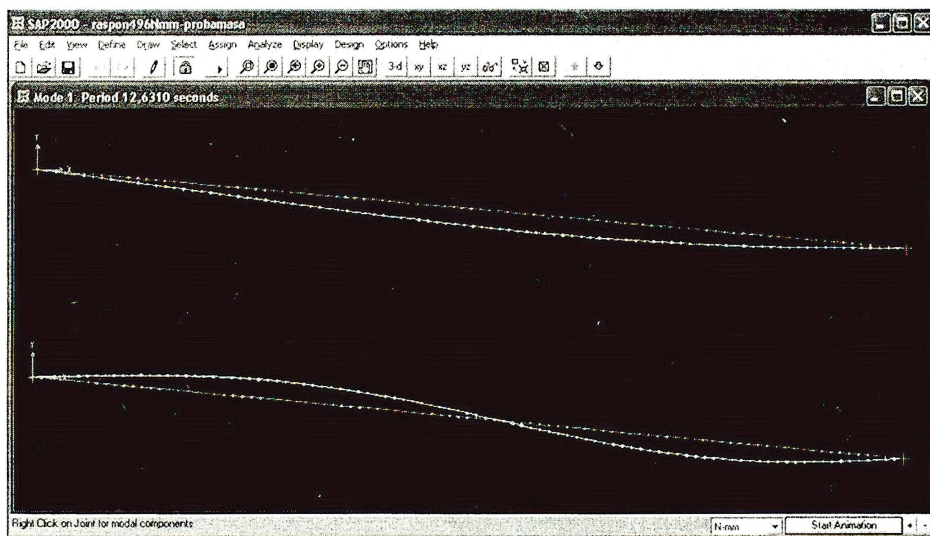


Figure 13 First and second mod of oscillation  
slika 13 Prvi i drugi oblik oscilovanja

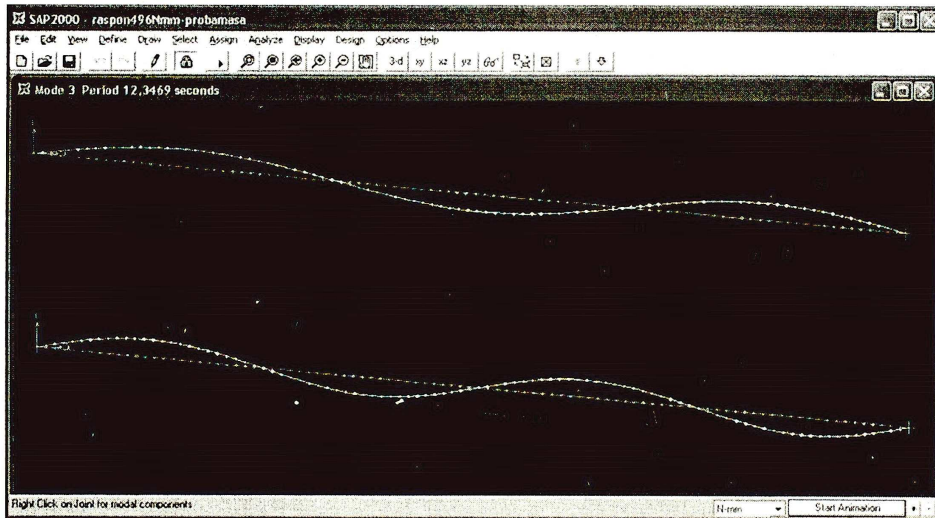


Figure 14 Third and fourth mod of oscillation  
slika 14 Treći i četvrti oblik oscilovanja

In Table 2 comparative values for the lowest natural frequencies of oscillation are given, measured analytically and by the method of final elements:

U tabeli 2 date su uporedne vrednosti za najniže kružne frekvencije oscilovanja, merenjem, analitički i metodom konačnih elemenata:

Table 2 Comparative values  
tabela 2 Uporedne vrednosti

Span l	$f_l$ [Hz]		
	measured	analytically	method of final elements
1	0.313	0.3132	0.323
2	0.257	0.2562	0.261
3	0.226	0.2248	0.23
4	0.239	0.2383	0.247

### 3 CONCLUSION

The lowest natural frequencies for all three modes differ very little. The difference is from 3 to 5%. What appears to be disadvantage of the software used for this kind of calculation is the fact that there is linear increase of natural frequencies for the following modes, which is not the case with measurement on a cableway. The problem is that the software operates with discrete values of concentrated masses, which means that the change of the mass due to the increase of the number of cable-cars cannot be taken into account.

### 3 ZAKLJUČAK

Najniže sopstvene kružne frekvencije se za sva tri načina veoma malo razlikuju. Razlika je od 3-5%. Ono što je mana softvera koji je korišćen za ovu vrstu proračuna je što imamo linearni porast sopstvenih kružnih frekvencija za naredne modove, a što kod merenja na žičari nije slučaj. Problem je u tome što softver radi sa diskretnim vrednostima koncentrisanih masa, tj. promenu mase usled povećanja broja kabina ne možemo uzeti u obzir.

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