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TRAPEZOID CROSS-SECTION OPTIMIZATION OF THE HOISTING DEVICE LATTICE STRUCTURES

OPTIMIZACIJA TRAPEZNOG POPREČNOG PRESEKA DIZALIČNIH REŠETKASTIH KONSTRUKCIJA

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Abstract: *This paper discusses the problem of trapezoid cross-section optimization of the hoisting device lattice structures. Minimal total mass of the construction has been selected as the objective function, while the criterion of stress was used as the constraint function. The optimization is achieved in two manners: by applying Lagrange's multipliers method and by usage of finite element method. The comparison of the obtained results was carried out on the example of the real hoisting device lattice construction. The suggested model can be easily modified for the application on similar lattice constructions which are parts of tower cranes and conveyor systems.*

Key words: *lattice structure, hoisting device, design optimization, Lagrange multipliers, finite elements method.*

Apstrakt: *Rad obrađuje problem optimizacije trapeznog poprečnog preseka dizaličnih rešetkastih konstrukcija. Za funkciju cilja je usvojena minimalna masa konstrukcije, dok je kao funkcija ograničenja usvojen kriterijum čvrstoće. Optimizacija je ostvarena na dva načina: primenom metode Lagranžovih množitelja i primenom metode konačnih elemenata. Poređenje dobijenih rezultata je ostvareno na primeru realne rešetkaste konstrukcije koja pripada dizalici. Predloženi model se uz manje modifikacije može primeniti i na druge rešetkaste konstrukcije koje pripadaju dizalicama i transportnim stazama.*

Ključne reči: *rešetkasta konstrukcija, dizalica, optimizacija, Lagranževi množitelji, konačni elementi.*

1 INTRODUCTION

The analysis of the cost structure of a metal construction, that was carried out by Farkas [3], is shown that the price is primarily influenced by the price of the material (30 - 73)%, while the other costs are lower: the manufacture (16 - 22)%, assembling (5 - 20)%, transportation (3 - 7)%, designing (2 - 3)%. The selection of optimum shape and optimum parameters of metal construction reduces the consumption of the material and its price.

1. UVOD

Analizom strukture cene metalnih konstrukcija Farkas [3] dolazi do zaključka da osnovni deo čini utrošak materijala (63 – 73%), dok su ostali uticaji znatno manji: izrade (16 – 22%), montaže (5 – 20%), transporta (3 – 7%), projektovanja (2 – 3%). Izborom optimalnog oblika i optimalnih parametara metalnih konstrukcija smanjuje se utrošak materijala i njihova cena.

The problem of construction optimization, using different methods, objective and constraint functions, was studied by several authors. Farkas [4, 5, 6] came to conclusion that the mathematical problems of the conditional extreme of the function of several variables can be applied onto structure cross-section in order to optimize it from the aspect of load and material consumption.

The optimization of the elastic rod for axial load case was achieved by implementation of Pontriyagin's maximum principle in paper [7]. The authors showed that the selection of the elastic rod optimal shape for certain cases of buckling can provide saving of the material up to 30%. The optimal dimensions of the cross sections shaped as regular polygons with "n" sides ($n \geq 3$) were determined in paper [2]. The optimization was carried out for adopted stress and deformation as the constraint functions, and minimal cross section area as the objective function. In paper [1] the problem of the optimization of thin-walled cantilever open section beams submitted to bending and limited torsion was analyzed.

The papers which are, in the first place, dealing with the optimization of the lattice structures, can be found in the available literature. As an example there can be mentioned the finite element optimization of the frameworks [6], optimization of the rectangular cross section of lattice structures in case where the criterion of stress was the constraint function [8], and the optimization of the triangular cross section of lattice structures, where both criteria of stress and deformation were used as constraint functions [9]. In paper [10], besides the determination of optimal dimensions of various cross sections for different supporters, theoretical bases of the optimization by application of Lagrange's multipliers method was analyzed.

The main objective of this paper is to give the solution to the problem of the trapezoid cross section optimization of the hoisting device lattice structures. The emphasis was set on hoisting device lattice structure which was modeled as cantilever beam with an overhang that carries the load trolley. In this case, in model is included practical data that the overall dimensions of cross section are constant along the lengthwise coordinate of the lattice structure. In order to

Problemom optimizacije konstrukcija različitim metodama, funckijama cilja i funkcijama ograničenja bavilo se više autora. Farkas [3,4,5] zaključuje da bi matematičke probleme uslovnog ekstremuma funkcije više promenljivih mogao da se primeni na poprečni presek konstrukcije u cilju optimizacije poprečnih preseka sa stanovišta opterećenja i utroška materijala.

Optimizacija elastičnog štapa za slučaj aksijalnog naprezanja primenom Pontriyaginovog principa maximuma je ostvarena u radu [7]. Autori su dokazali da se izborom optimalnog oblika elastičnog štapa za određene slučajeve izvijanja može ostvariti ušteda materijala i do 30%. Optimalne dimenzije poprečnih preseka oblika pravilnih poligona sa "n" stranica ($n \geq 3$) su određene u radu [2]. Optimizacija je urađena za usvojenu čvrstoću i deformaciju kao funkcije ograničenja i minimalnu površinu poprečnog preseka kao funkciju cilja. Radom [1] je rešavan problem optimizacije tankozidih konzolnih konstrukcionih elemenata otvorenih poprečnih preseka izloženih savijanju i ogranicenoj torziji.

U literaturi se mogu naći radovi koji se prvenstveno bave optimizacijom rešetkastih konstrukcija. Kao primer se može navesti optimizacija rešetkastih ramova metodom konačnih elemenata [6], optimizacija pravougaonog poprečnog preseka rešetkastih konstrukcija za slučaj funkcije ograničenja u obliku kriterijuma čvrstoće [8], kao i određivanje optimalnih parametara trougaonog poprečnog preseka rešetkastih konstrukcija za slučaj čvrstoće i deformacije kao funkcija ograničenja [9]. U radu [10] se osim definisanja optimalnih dimenzija nosača različitih poprečnih preseka definišu i teorijske osnove optimizacije primenom metode Lagranžovig množitelja.

Cilj ovog rada je da pruži rešenje problema optimizacije trapeznog poprečnog preseka dizaličnih rešetkastih kontsrukcija. Akcenat je stavljen na analizu dizaličnih rešetkastih konstrukcija po kojima se kreću kolica sa teretom. Kao model je u vertikalnoj ravni usvojen gredni nosač sa prepustom. Ovakav model je veoma rasprostranjen u slučaju rešavanja pretovara tereta različite namene. U ovom slučaju u model je uključen realan podatak koji nas upućuje na činjenicu da se gabaritne dimenzije poprečnog preseka većine rešetkastih konstrukcija ne menjaju duž njene

satisfy requirement that maximum stress in any given cross section should be equal to allowable stress, here is suggested that the cross section area of flanges is variable along the lengthwise coordinate of the lattice structure.

2 DEFINING OF THE OPTIMIZATION TASK

Lattice constructions have important applications in practice, especially in the case of the constructions with larger overall dimensions of which is required larger carrying capacity and minimal weight. The hoisting device assembly and frameworks can be mentioned as the examples of those constructions, as well as constructions which are the integral part of building constructions.

The cross section of lattice construction (figure 1) consists of flanges (positions 1-4) and lattice (positions 5-8). The flanges are usually made of pipes with circular, square or rectangular cross section. They can also be made of "L" or "U" profiles combination.

podužne ose. Kako bi se zadovoljio zahtev da maksimalni napon u bilo kom poprečnom preseku bude jednak dopuštenom naponu, predloženo je da se površina poprečnog preseka pojaseva menja duž podužne ose konstrukcije.

2 DEFINISANJE OPTIMIZACIONOG ZADATKA

Rešetkaste konstrukcije imaju značajnu primenu u praksi i to naročito kod konstrukcija većih dimenzija od kojih se zahteva veća nosivost i manja sopstvena masa. Kao primer se mogu navesti sklopovi dizalica, kao i konstrukcije koje čine sastavni deo građevinskih objekata.

Poprečni presek rešetkaste konstrukcije (slika 1) čine pojasevi (pozicije 1-4) i rešetkasta ispuna (pozicije 5-8). Pojasevi su najčešće izrađeni od cevi kružnog, kvadratnog ili pravougaonog poprečnog preseka ili od kombinacije "L" ili "U" profila.

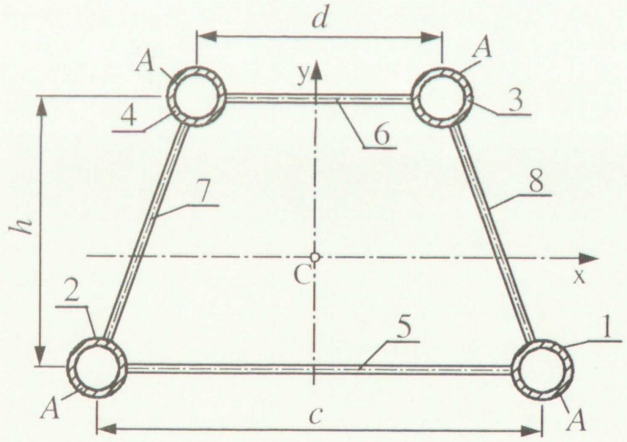


Figure 1 Trapezoid cross section
slika 1 Trapezni poprečni presek

The analysis of the lattice structures is shown that the most used are those with triangular, rectangular and trapezoid cross section. The recommendations in available literature give basic dimensions of lattice construction cross section in function of its length. Still, they ignore many other influential factors, such as loads, material ..

As the result of the trapezoid cross section optimization of the lattice structures, the following optimal parameters are obtained:

Analizom realnih rešetkastih konstrukcija nalazimo da se najčešće primenjuju rešetkaste konstrukcije sa trougaonim, pravougaonim i trapeznim poprečnim presecima. Postojeće preporuke u literaturi daju osnovne dimenzije poprečnog preseka rešetkastih konstrukcija u funkciji dužine konstrukcije. Ovim preporukama se zanemaruju mnogi drugi uticajni faktori kao što su opterećenja, materijal...

Kao rezultat optimizacije trapeznog poprečnog preseka rešetkastih konstrukcija dobijaju se sledeći optimalni parametri:

- c, d, h – dimensions of trapezoid cross section (figure 1),
- A – cross section area of one flange.

The dimensions of trapezoid cross section (c, d, h) are constant along the structure, while the cross section area of flange is variable.

- c, d, h – dimenzije trapeznog poprečnog preseka (slika 1),
- A – površina poprečnog preseka jednog pojasa.

Geometrijska rastojanja poprečnog preseka (c, d, h) uzimaju konstantne vrednosti duž nosača, dok se površina menja.

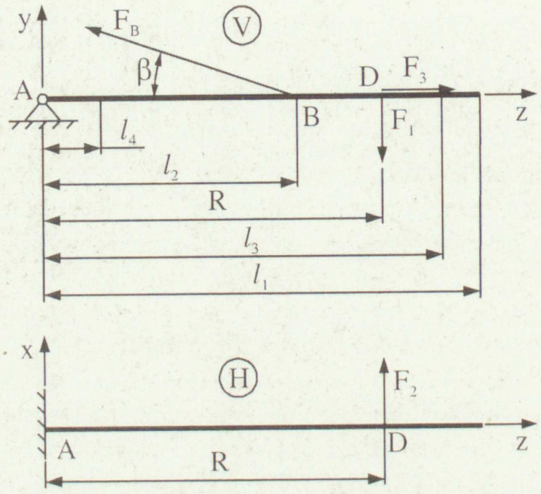


Figure 2. Static model
slika 2. Statički model

The optimal dimensions of trapezoid cross section are influenced by the type of the supporter and by the type of the applied loads. Earlier established assumption was that the dimensions of trapezoid cross section (c, d, h) are constant along the lengthwise coordinate of the lattice structure. So, it is impossible to obtain expressions which can provide optimal parameters for all type of supporters. In practice, the supporters are usually modeled as the beam with an overhang and as the cantilever beam. Therefore, the static model shown in the figure 2 was analyzed in the following text, for the case when the load is applied on the overhang ($l_2 \leq R \leq l_3$).

3 MATHEMATICAL MODEL

Vector of given parameters is:

$$\vec{x} = (N, M_x, M_y, [\sigma], \ldots) \qquad (1)$$

where: N is the axial force, acting on the cross-section center,
M_x, M_y are the moments of flexion for axes x and y, respectively,
$[\sigma]$ is the limiting (allowable) stress,

Optimalne dimenzije trapeznog poprečnog preseka zavise od tipa nosača i opterećenja. Kako smo uveli pretpostavku da geometrijska rastojanja poprečnog preseka (c, d, h) uzimaju konstantne vrednosti duž nosača to je nemoguće definisati izraze čijim korišćenjem će se dobiti vrednosti optimalnih parametara za sve tipove nosača. Kako se u praksi veoma često mogu naći nosači koji se modeliraju gredom sa prepustom i konzolom, to će se u nastavku razmatrati statički model prikazan na slici 2 i to za slučaj kada se teret nalazi na prepustu ($l_2 \leq R \leq l_3$).

3 MATEMATIČKI MODEL

Vektor zadatih parametara je dat kao:

$$\vec{x} = (N, M_x, M_y, [\sigma], \ldots) \qquad (1)$$

gde je: N - aksijalna sila, koja deluje u centru poprečnog preseka,
M_x, M_y - momenti savijanja za x i y, redom,
$[\sigma]$ - dozvoljeni (granični) napon.

Vector of variables is given by:

$$\vec{y} = (c, d, h, ...). \quad (2)$$

Constrain function is presented as:

$$\varphi_1 = \varphi_1(c, d, h, ...), \quad \varphi_2 = \varphi_2(c, d, h, ...), \ ... \quad (3)$$

The determination of optimum parameters was performed by the method of Lagrange multipliers for extremes of the function of several variables. Therefore, for the function $F=F(c, d, h, ...)$ to have relative maximum or minimum at a certain point, it is necessary that constrain function (3) be satisfied, as well as the following equations:

$$\frac{\partial \Phi}{\partial a} = 0, \ \frac{\partial \Phi}{\partial b} = 0, \ \frac{\partial \Phi}{\partial h} = 0, \ ... \quad (4)$$

where Lagrange's function is:

$$\Phi(c,d,h,...) = F(c,d,h,...) + \lambda_1 \cdot \varphi_1(c,d,h,...) + \\ + \lambda_2 \cdot \varphi_2(c,d,h,...) + ... \quad (5)$$

and λ – unknown Lagrange multipliers. So, the system of equations (4) can be expressed as:

$$\frac{\partial F}{\partial a} + \lambda_1 \frac{\partial \varphi_1}{\partial a} + \lambda_2 \frac{\partial \varphi_2}{\partial a} + ... = 0, \ \frac{\partial F}{\partial b} + \lambda_1 \frac{\partial \varphi_1}{\partial b} + \lambda_2 \frac{\partial \varphi_2}{\partial b} + ... = \\ = 0, \ \frac{\partial F}{\partial h} + \lambda_1 \frac{\partial \varphi_1}{\partial h} + \lambda_2 \frac{\partial \varphi_2}{\partial h} + ... = 0, ... \quad (6)$$

The system of equations (3) and (6) defines unknown optimum parameters (c_O, d_O, h_O, ...) and unknown Lagrange multipliers λ.

4 CONSTRAINT FUNCTION

The stress criterion was used as constraint function. According to this criterion, the constraint function is based on the condition that for all values of coordinate (z) maximum stress which occurs in one of the flanges of the construction should be equal to allowable stress:

$$\varphi(z) = \left| \frac{N(z)}{4A} + \frac{M_x(z)}{I_x} \cdot y + \frac{M_y(z)}{I_y} \cdot x \right| - [\sigma] = 0, \quad (7)$$

By substituting the expressions for the moments of inertia of trapezoid cross section (I_x and I_y), the constraint function based on the criterion of stress is obtained:

Vektor promenljivih je dat kao:

$$\vec{y} = (c, d, h, ...). \quad (2)$$

Funkcija ograničenja je predstavljena kao:

$$\varphi_1 = \varphi_1(c, d, h, ...), \quad \varphi_2 = \varphi_2(c, d, h, ...), \ ... \quad (3)$$

Određivanje optimalnih parametara izvedeno je metodom Lagranževih množitelja za eksteme funkcija sa više promenljivih. Znači, da bi funkcija $F=F(c,d,h,...)$ imala relativni minimum ili maksimum u nekoj tački, potrebno je da bude zadovoljena ne samo funkcija ograničenja (3), već i sledeće jednačine:

$$\frac{\partial \Phi}{\partial a} = 0, \ \frac{\partial \Phi}{\partial b} = 0, \ \frac{\partial \Phi}{\partial h} = 0, \ ... \quad (4)$$

gde je Lagranževa funkcija:

$$\Phi(c,d,h,...) = F(c,d,h,...) + \lambda_1 \cdot \varphi_1(c,d,h,...) + \\ + \lambda_2 \cdot \varphi_2(c,d,h,...) + ... \quad (5)$$

a λ – nepoznati Lagranževi množitelji. Dakle, sistem jednačina (4) se može predstaviti kao:

$$\frac{\partial F}{\partial a} + \lambda_1 \frac{\partial \varphi_1}{\partial a} + \lambda_2 \frac{\partial \varphi_2}{\partial a} + ... = 0, \ \frac{\partial F}{\partial b} + \lambda_1 \frac{\partial \varphi_1}{\partial b} + \lambda_2 \frac{\partial \varphi_2}{\partial b} + ... = \\ = 0, \ \frac{\partial F}{\partial h} + \lambda_1 \frac{\partial \varphi_1}{\partial h} + \lambda_2 \frac{\partial \varphi_2}{\partial h} + ... = 0, ... \quad (6)$$

Sistem jednačina (3) i (6) određuje nepoznate optimalne parametre (c_O, d_O, h_O, ...) i nepoznati Lagranžov množitelj λ.

4 FUNKCIJA OGRANIČENJA

Za funkciju ograničenja usvojen je kriterijum čvrstoće. Prema kriterijumu čvrstoće funkcija ograničenja se definiše na osnovu uslova da je za sve vrednosti podužne koordinate (z) maksimalni stvarni napon koji se javlja u jednom od pojaseva jednak dopuštenom:

$$\varphi(z) = \left| \frac{N(z)}{4A} + \frac{M_x(z)}{I_x} \cdot y + \frac{M_y(z)}{I_y} \cdot x \right| - [\sigma] = 0, \quad (7)$$

gde zamenom izraza za momente inercije trapeznog poprečnog preseka (I_x i I_y) sledi funkcija ograničenja na osnovu kriterijuma čvrstoće:

$$\varphi(z) = \frac{1}{A}\left|\frac{N(z)}{4} + \frac{M_x(z)}{h^2}\cdot y + \frac{2\cdot M_y(z)}{c^2+d^2}\cdot x\right| - [\sigma] = 0 \quad (8)$$

The model of a lattice structure was analyzed as a static model in two different planes: in vertical plane it is pin connected beam (at A), supported by roller support (at B); in horizontal plane it is modeled as fixed supported beam (figure 2) [9,10], where:

F_1 – is the resultant force acting in vertical plane at range R due to the dead load and trolley weight,
F_2 – is the force acting in horizontal plane at the range R, a tangential force due to construction rotation (if the construction is capable for this kind of motion),
F_3 – is the force acting in the direction of lengthwise axis at the range R, a centrifugal force due to the construction rotation,
R – is the range, which determines the load position in respect of joint A,

The analysis of the available hoisting device structures reveals different design options for support B. The cable support was considered in this analysis.

The value of load that hoisting device construction can carry depends on the range R, in most degree. In order to analyze the real lattice structure, the following carrying capacity was used in this paper[9]:

$$Q = \left(428.861 - 41.106\sqrt{R}\right)^2, \ [N], \quad (9)$$

where R is in meters.

The static model (figure 2) can be applied for the determination of optimal design parameters of conveyor belts and other conveyor system, with minor modifications. In those cases, distributed loading should be applied on the supporter in vertical plane. If the conveyor belt is designating for people transportations, the several forces should be applied, acting on the most critical positions for the construction. One of that positions will be the end of the construction, for sure.

The variations of axial force and bending moments can be determined, based on static model (figure 2), as functions of range (R) and lengthwise coordinate (z).

$$\varphi(z) = \frac{1}{A}\left|\frac{N(z)}{4} + \frac{M_x(z)}{h^2}\cdot y + \frac{2\cdot M_y(z)}{c^2+d^2}\cdot x\right| - [\sigma] = 0 \quad (8)$$

Model konstrukcije je analiziran kao statički model u dve različite ravni: u vertikalnoj ravni modelira se gredom koja je oslonjena na jedan nepokretan (tačka A) i jedan pokretan oslonac (tačka B), dok se u horizontalnoj može modelirati kao konzola (slika 2) [9,10], gde su:

F_1 – rezultujuća sila koja deluje u vertikalnoj ravni na dohvatu R, a čine je težine tereta i kolica,
F_2 – rezultujuća sila koja deluje u horizontalnoj ravni na dohvatu R, a čini je tangencijalna sila usled obrtanja konstrukcije (ukoliko dizalica može da ostvari dato kretanje),
F_3 – rezultujuća sila koja deluje u pravcu podužne ose na dohvatu R, a čini je centrifugalna sila usled obrtanja konstrukcije,
R – dohvat kojim se definiše položaj tereta u odnosu na oslonac A,

Analizom raspoloživih dizaličnih konstrukcija mogu se uočiti različita konstruktivna rešenja oslonca B. U ovoj analizi će se posmatrati slučaj užeta.

Numerička vrednost tereta kojim se može opteretiti dizalična rešetkasta konstrukcija u najvećoj meri zavisi od dohvata. Kako bi smo analizirali realnu konstrukciju to smo u nastavku koristili sledeću karakteristiku nosivosti [9]:

$$Q = \left(428.861 - 41.106\sqrt{R}\right)^2, \ [N], \quad (9)$$

gde se dohvat izražava u metrima.

Statički model (slika 2) uz manje modifikacije se može primeniti i za slučaj određivanja optimalnih dimenzija transportnih staza. Ukoliko one čine noseću konstrukciju transportera potrebno je nosač u vertikalnoj ravni opteretiti kontinualnim opterećenjem. U slučaju da je transportna staza namenjena transportu ljudi potrebno je postaviti koncentrisana opterećenja u najnepovoljnijim pozicijama po konstrukciju. Svakako, jedna od njih će biti vrh konstrukcije.

Na osnovu statičkog modela (slika 2) mogu se definisati promene aksijalne sile i momenata savijanja u funkciji od dohvata (R) i podužne koordinate (z).

By the application of expression (8), a correspondence between flange cross section area, load, allowable stress and dimensions of trapezoid cross section, can be written as a next function:

$$A = \frac{1}{[\sigma]}\left|\frac{N(z)}{4} + \frac{M_x(z)}{h^2}\cdot y + \frac{2\cdot M_y(z)}{c^2+d^2}\cdot x\right|. \quad (10)$$

As it is assumed that the dimensions of trapezoid cross section (c, d, h) have constant values along the construction, it can be concluded that, based on the analysis of the expression (10-12), the cross section area of flanges are dependent of the axial force and bending moment's variations along the construction. Static model of the construction (figure 2) is subjected to concentrated forces (F_1, F_2, F_3). This causes linear change of bending moments with lengthwise coordinate "z". The configuration of this linear function changes with the position of constrains and loads. Based on this, it follows that cross section area of flanges vary linearly with lengthwise coordinate "z", i.e., the equation (12) can be written as:

$$A(z) = \begin{cases} a_{10} + a_{11}\cdot z & for \quad 0 \le z \le l_2 \\ a_{20} + a_{21}\cdot z & for \quad l_2 < z \le R \\ 0 & for \quad R < z \le l_1 \end{cases} \quad (11)$$

Finally, in correspondence with load position, the optimal parameters of trapezoid cross section of the lattice construction are determined, in following text:

c, d, h – dimensions of trapezoid cross section,
a_{10}, a_{11}, a_{20}, a_{21} – parameters which determine cross section area of flanges.

5 OBJECTIVE FUNCTION

As it was mentioned before, the price of the material, hence, material consumption has the main influence in price of metal construction overall. Hence, total mass of the construction has been selected as the objective function:

$$F \equiv m = \rho \cdot V, \quad (12)$$

where: ρ – is density of the material,
V – volume of the lattice construction.

Koristeći izraz (8) može se napisati funkcija zavisnosti površine poprečnog preseka pojasa od opterećenja, dopuštenog napona i geometrijskih rastojanja poprečnog preseka:

$$A = \frac{1}{[\sigma]}\left|\frac{N(z)}{4} + \frac{M_x(z)}{h^2}\cdot y + \frac{2\cdot M_y(z)}{c^2+d^2}\cdot x\right|. \quad (10)$$

Kako je uvedena pretpostavka da geometrijska rastojanja poprečnog preseka (c, d, h) uzimaju konstantne vrednosti duž konstrukcije, to se analizom izraza (10-12) može zaključiti da površina poprečnog preseka pojaseva zavisi od oblika promene aksijalne sile i momenata savijanja duž konstrukcije. Statički model (slika 2) opterećuju koncentrisane sile (F_1, F_2, F_3) što prouzrokuje linearnu zavisnost momenata savijanja od podužne koordinate "z", čija se funckija zavisnosti menja na mestima dejstva sila i mestima gde se nalaze oslonci. Na osnovu prethodnih zapažanja sledi da je površina poprečnih preseka pojaseva u linearnoj vezi za podužnom koordinatom konstrukcije (z), tj. da se izraz (12) može zapisati u sledećem obliku:

$$A(z) = \begin{cases} a_{10} + a_{11}\cdot z & za \quad 0 \le z \le l_2 \\ a_{20} + a_{21}\cdot z & za \quad l_2 < z \le R \\ 0 & za \quad R < z \le l_1 \end{cases} \quad (11)$$

Konačno, u nastavku su određeni sledeći optimalni parametri trapeznog poprečnog preseka rešetkaste konstrukcije:

c, d, h – dimenzije poprečnog preseka,
a_{10}, a_{11}, a_{20}, a_{21} – parametri kojima se definiše površina poprečnog preseka pojaseva.

5 FUNKCIJA CILJA

Kao što je prethodno pomenuto, utrošak materijala ima značajno učešće u ukupnoj ceni metalnih konstrukcija. Stoga je za funkciju cilja usvojena minimalna masa konstrukcije:

$$F \equiv m = \rho \cdot V, \quad (12)$$

gde je: ρ – gustina materijala,
V – zapremina rešetkaste konstrukcije.

Volume of the lattice construction is equal to the sum of flanges volume (V_p), volume of lattice (V_r) and volume of other elements (V_o):

$$V = V_p + V_r + V_o. \quad (13)$$

The volume of the flanges is determined by next expressions:

$$V_p = 4 \cdot \int_0^{L_1} A dz = 4 \cdot \int_0^{L_2} a_{10} + a_{11} \cdot z \, dz + 4 \cdot \int_{L_2}^{R} a_{20} + a_{21} \cdot z \, dz \quad (14)$$

After integration expression (14) becomes

$$V_p = 4 \cdot a_{10} \cdot L_2 + 2 \cdot a_{11} \cdot L_2^2 + 4 \cdot a_{20} \cdot (R - L_2) + 2 \cdot a_{21} \cdot \left(R^2 - L_2^2\right) \quad (15)$$

Volume of lattice is determined by:

$$V_r = k_1 (c + d) + k_2 \sqrt{4h^2 + (c - d)^2}, \quad (16)$$

while volume of other elements is determined by following expression

$$V_o = k_3 (V_p + V_r). \quad (17)$$

The variation of coefficients values k_1, k_2, k_3 is minimal in observed cases, so they can be assumed to be constant [3,4,5,9,10].

Substituting the expression (17) in (13) provide the expression which determine volume of lattice construction

$$V = (1 + k_3) \cdot (V_p + V_r). \quad (18)$$

6 OPTIMAL PARAMETERS

Lagrange's function (5) is given as:

$$\Phi = V + \lambda_1 \cdot \varphi(z_1) + \lambda_2 \cdot \varphi(z_2) + \lambda_3 \cdot \varphi(z_3) + \lambda_4 \cdot \varphi(z_4) \quad (19)$$

where: $\varphi(z_i)$ represent constraint function based on the criterion of stress, written for the following values of lengthwise coordinates:

$0 \le z_1 \le l_2$, $0 \le z_2 \le l_2$, $l_2 \le z_3 \le R$, $l_2 \le z_4 \le R$.

After substituting expressions (8), (18), (19) in expression (4) the systems of equations are obtained. Solutions of these systems of equations

Zapremina rešetkaste konstrukcije je jednaka zbiru zapremine pojaseva (V_p), zapremine rešetkaste ispune (V_r) i zapremine ostalih elemenata (V_o):

$$V = V_p + V_r + V_o. \quad (13)$$

Zapremina pojaseva se određuje sledećim izrazima:

$$V_p = 4 \cdot \int_0^{L_1} A dz = 4 \cdot \int_0^{L_2} a_{10} + a_{11} \cdot z \, dz + 4 \cdot \int_{L_2}^{R} a_{20} + a_{21} \cdot z \, dz \quad (14)$$

odakle se posle izvršene integracije dobija

$$V_p = 4 \cdot a_{10} \cdot L_2 + 2 \cdot a_{11} \cdot L_2^2 + 4 \cdot a_{20} \cdot (R - L_2) + 2 \cdot a_{21} \cdot \left(R^2 - L_2^2\right) \quad (15)$$

Zapremine rešetkaste ispune je određena sa:

$$V_r = k_1 (c + d) + k_2 \sqrt{4h^2 + (c - d)^2}, \quad (16)$$

dok se zapremine ostalih elemenata određuje primenom sledećih izraza

$$V_o = k_3 (V_p + V_r). \quad (17)$$

Promena vrednosti koeficijenata k_1, k_2, k_3 je u posmatranim slučajevima mala pa se mogu smatrati konstantnim veličinama [3,4,5,9,10].

Zamenom izraza (17) u (13) dobija se izraz kojim se definiše zapremina rešetkaste konstrukcije

$$V = (1 + k_3) \cdot (V_p + V_r). \quad (18)$$

6 OPTIMALNE DIMENZIJE

Lagranževa funkcija (5) je oblika:

$$\Phi = V + \lambda_1 \cdot \varphi(z_1) + \lambda_2 \cdot \varphi(z_2) + \lambda_3 \cdot \varphi(z_3) + \lambda_4 \cdot \varphi(z_4) \quad (19)$$

gde su sa $\varphi(z_i)$ označene funkcije ograničenja na osnovu kriterijuma čvrstoće (8) napisane za sledeće vrednosti podužnih koordinata:

$0 \le z_1 \le l_2$, $0 \le z_2 \le l_2$, $l_2 \le z_3 \le R$, $l_2 \le z_4 \le R$.

Posle zamene izraza (8), (18), (19) u izraz (4) dobija se sistem jednačina na osnovu kojih se određuju nepoznati parametri zavisno od položaja

determine unknown parameters as functions of load position and position of the flange in which maximal stress occurs. There are two possible combinations of position of the flange in which maximal stress occurs:

tereta i položaja pojasa kod kog se javlja maksimalni stvarni napon. Postoje dve moguće kombinacije položaja pojasa u kome se javlja maksimalni napon:

- maximal actual stress occurs in one of the lower flanges (figure 1 – position 1 and 2)

- maksimalni stvarni napon se javlja u jednom od donjih pojaseva (slika 1 – pozicije 1 i 2)

$$k_1 + k_2 \cdot \frac{c-d}{\sqrt{4\cdot h^2 + (c-d)^2}} = \frac{c^2 - d^2}{(c^2+d^2)^2} \cdot \frac{2\cdot F_2 \cdot R^2}{[\sigma]}, \qquad \frac{h^3}{\sqrt{4\cdot h^2 + (c-d)^2}} = \frac{F_1 \cdot R \cdot (R-l_2)}{4\cdot k_2 \cdot [\sigma]},$$

$$2\cdot c\cdot d\cdot\left(k_1 + k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right) = \left(c^2-d^2\right)\cdot\left(k_1 - k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right)$$

$$a_{10} = \frac{1}{[\sigma]}\left[\frac{1}{4}\cdot\left(F_1\frac{R}{l_2}ctg\beta - F_3\right) + \frac{F_2\cdot R\cdot c}{c^2+d^2}\right], \qquad a_{11} = \frac{1}{[\sigma]}\left[\frac{F_1\cdot(R-l_2)}{2\cdot h\cdot l_2} - \frac{F_2\cdot c}{c^2+d^2}\right]$$

$$a_{20} = \frac{1}{[\sigma]}\left[\frac{-F_3}{4} + \frac{F_1\cdot R}{2\cdot h} + \frac{F_2\cdot R\cdot c}{c^2+d^2}\right], \qquad a_{21} = \frac{1}{[\sigma]}\left[-\frac{F_1}{2\cdot h} - \frac{F_2\cdot c}{c^2+d^2}\right]; \tag{20}$$

- maximal actual stress occurs in one of the upper flanges (figure 1 – position 3 and 4)

- maksimalni stvarni napon se javlja u jednom od gornjih pojaseva (slika 1 – pozicije 3 i 4)

$$k_1 + k_2 \cdot \frac{c-d}{\sqrt{4\cdot h^2 + (c-d)^2}} = \frac{4\cdot c\cdot d\cdot F_2\cdot R^2}{(c^2+d^2)^2\cdot[\sigma]}, \qquad \frac{h^3}{\sqrt{4\cdot h^2 + (c-d)^2}} = \frac{F_1\cdot R\cdot(R-l_2)}{4\cdot k_2\cdot[\sigma]},$$

$$2\cdot c\cdot d\cdot\left(k_1 - k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right) = \left(d^2-c^2\right)\cdot\left(k_1 + k_2\cdot\frac{c-d}{\sqrt{4\cdot h^2+(c-d)^2}}\right),$$

$$a_{10} = \frac{1}{[\sigma]}\left[\frac{-1}{4}\cdot\left(F_1\frac{R}{l_2}ctg\beta - F_3\right) + \frac{F_2\cdot R\cdot d}{c^2+d^2}\right], \qquad a_{11} = \frac{1}{[\sigma]}\left[\frac{F_1\cdot(R-l_2)}{2\cdot h\cdot l_2} - \frac{F_2\cdot d}{c^2+d^2}\right]$$

$$a_{20} = \frac{1}{[\sigma]}\left[\frac{F_3}{4} + \frac{F_1\cdot R}{2\cdot h} + \frac{F_2\cdot R\cdot d}{c^2+d^2}\right], \qquad a_{21} = \frac{1}{[\sigma]}\left[-\frac{F_1}{2\cdot h} - \frac{F_2\cdot d}{c^2+d^2}\right]; \tag{21}$$

The conditions that determine in which flange will occur maximal stress for any given lengthwise coordinate "z", are defined with expression (8). Therefore, the maximal stress will occur in one of the lower flanges (figure 1 – position 1 and 2), corresponding the load positions, if the following conditions are met:

Uslovi na osnovu kojih se može odgovoriti na pitanje u kom se pojasu se za bilo koju vrednost podužne koordinate "z" javlja maksimalni napon se definišu na osnovu izraza (8), pa će se tako maksimalni napon javiti u jednom od donjih pojaseva (slika 1 – pozicije 1 i 2) zavisno od položaja tereta ukoliko su ispunjeni sledeći uslovi:

- $0 \le z \le l_2$

$$\left|\frac{1}{4}\cdot\left(F_1\cdot\frac{R}{l_2}\cdot ctg\beta - F_3\right) + \frac{F_1\cdot(R-l_2)\cdot z}{2\cdot h\cdot l_2} + \frac{F_2\cdot(R-z)\cdot c}{c^2+d^2}\right| >$$

$$> \left|\frac{-1}{4}\cdot\left(F_1\cdot\frac{R}{l_2}\cdot ctg\beta - F_3\right) + \frac{F_1\cdot(R-l_2)\cdot z}{2\cdot h\cdot l_2} + \frac{F_2\cdot(R-z)\cdot d}{c^2+d^2}\right|, \tag{22}$$

– $l_2<z\leq R$

$$\left|\frac{-F_3}{4}+\frac{F_1\cdot(R-z)}{2\cdot h}+\frac{F_2\cdot(R-z)\cdot c}{c^2+d^2}\right|>\left|\frac{F_3}{4}+\frac{F_1\cdot(R-z)}{2\cdot h}+\frac{F_2\cdot(R-z)\cdot d}{c^2+d^2}\right|, \quad (23)$$

7 NUMERICAL EXAMPLE AND ANALYSIS OF RESULTS

The verification of the theoretical results has been performed by the numerical example of the real hoisting device lattice construction, with following data:

- **limiting stress [σ] =240 MPa,**
- **k_1=0,078, k_2=0,062, k_3=0,05,**
- **trolley weight – 3.2 kN,**
- **coefficient of the drive class 1.05,**
- **dynamic coefficient 1.15,**
- **angular speed of the construction 0.084 s^{-1},**
- **angular acceleration of the construction 0.017 s^{-2},**
- **angle between the construction and the rope β=10°,**
- **geometrical parameters l_1=50 m, l_2=24 m, l_3=44,8 m, l_4=7,5 m.**

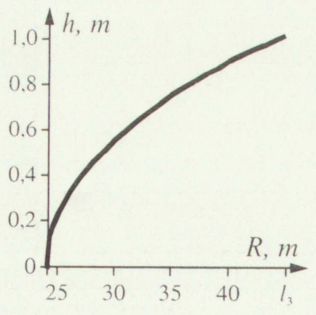
The centrifugal force due to load rotation (F_3) is axial force acting on a construction gantry. Numerical analysis of actual exploitation conditions showed that this force is less then 0,4% of axial force acting on a construction between points A and B.

7 NUMERIČKI PRIMER I ANALIZA REZULTATA

Verifikacija teorijskih rezultata je ostvarena na numeričkom primeru realne dizalične rešetkaste konstrukcije, a sa sledećim podacima:

- **dozvoljeni napon [σ] =240 MPa,**
- **k_1=0,078, k_2=0,062, k_3=0,05,**
- **težina kolica – 3.2 kN,**
- **koeficijent pogonske grupe 1.05,**
- **dinamički koeficijent 1.15,**
- **ugaona brzina konstrukcije 0.084 s^{-1},**
- **ugaono ubrzanje konstrukcije 0.017 s^{-2},**
- **ugao između konstrukcije i užeta β=10°,**
- **geometrijski parametri l_1=50 m, l_2=24 m, l_3=44,8 m, l_4=7,5 m.**

Centrifugalna sila usled obrtanja tereta (F_3) čini aksijalnu silu koja deluje na prepustu. Numeričkom analizom praktičnih eksploatacionih uslova može se zaključiti da je ona manja od 0,4% vrednosti aksijalne sile koja deluje na rasponu između mesta veze konstrukcije i stuba, i mesta veze konstrukcije i užeta.



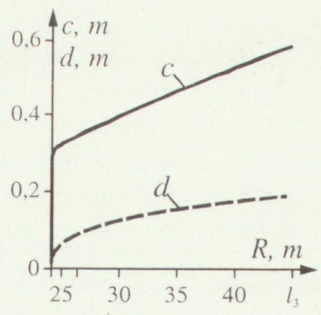


Figure 3 Variation of optimal dimensions of trapezoid cross section (c, d, h) with the range (R),

slika 3 Promena optimalnih vrednosti dimenzija poprečnog preseka (c, d, h) u funkciji od dohvata (R)

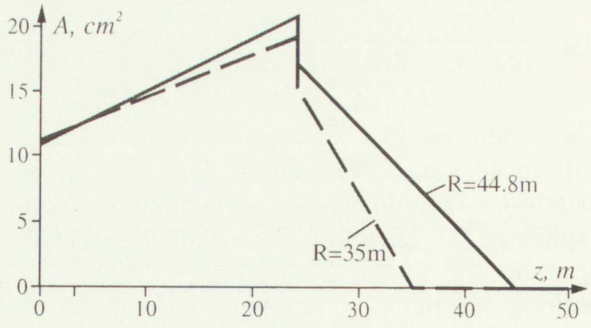


Figure 4 Variation of optimal values of cross section flange area with the lengthwise coordinate (z) for different values of range (R)
slika 4 Promena optimalnih vrednosti površine poprečnog preseka pojasa u funkciji podužne koordinate (z) za različite vrednosti dohvata (R)

The analysis of the variation of optimal dimensions of construction cross section along the range (figure 3), it can be concluded that trapezoid cross section is the optimal one. So, when the load is on construction gantry ($l_2 \leq R \leq l_3$) the actual maximal stress (pressure stress) would occur in one of the lower flanges, depending of the direction of hoisting device construction rotation. In that case the trapezoid cross section will be placed so that $c>d$.

The analysis of the variation of optimal values of cross section flanges area with the lengthwise coordinate (figure 4), it can be concluded that maximal values of those areas occurs in range of 30%. On the joint position B, the variation of cross section flanges area can be explained with the fact that the axial force acting on position ($0 \leq z \leq R$) is considerably grater then the force acting on position $R<z \leq l_2$.

8 OPTIMIZATION BY COMPUTER PROGRAM BASED ON FINITE ELEMENT ANALYSIS

The optimization of previous example was run by computer program based on finite element analysis with built in routines for design optimization. Those routines were first modified and then used for optimization of analyzed model in order to additionally verify application of obtained theoretical results in actual exploitation (figure 5).

Analizom promene optimalnih dimenzija poprečnog preseka u funkciji od dohvata (slika 3) može se zaključiti da trapezni poprečni presek predstavlja optimalni poprečni presek. Kada se teret nalazi na prepustu ($l_2 \leq R \leq l_3$) maksimalni stvarni napon (napon pritiska) će se, u zavisnosti od smera obrtanja dizalične konstrukcije, javiti u jednom od donjih pojaseva pa će u tom slučaju trapezni poprečni presek biti postavlje tako da je $c>d$

Analizom promene optimalnih vrednosti površine poprečnog preseka pojasa u funkciji podužne koordinate (slika 4) može se zaključiti da se maksimalne vrednosti optimalnih površina poprečnog preseka pojasa nalaze u rasponu od 30%. Na mestu tačke B se javlja promena optimalne površine poprečnog preseka pojasa što se objašnjava činjenicom da na rasponu ($0 \leq z \leq R$) deluje znatno veća aksijalna sila u poređenju sa rasponom $R<z \leq l_2$.

8 OPTIMIZACIJA RAČUNARSKIM PROGRAMOM BAZIRANIM NA METODI KONAČNIH ELEMENATA

Za isti primer je optimizacija datih parametara sprovedena i kompjuterskim programom, koji je baziran na metodi konačnih elemenata, a koji sadrži gotove rutine za optimizaciju konstrukcija. Ove rutine su modifikovane i iskorišćene za optimizaciju analilziranog modela da bi se dodatno opravdala primena dobijenih teorijskih rezultata u realnoj eksploataciji. (slika 5).

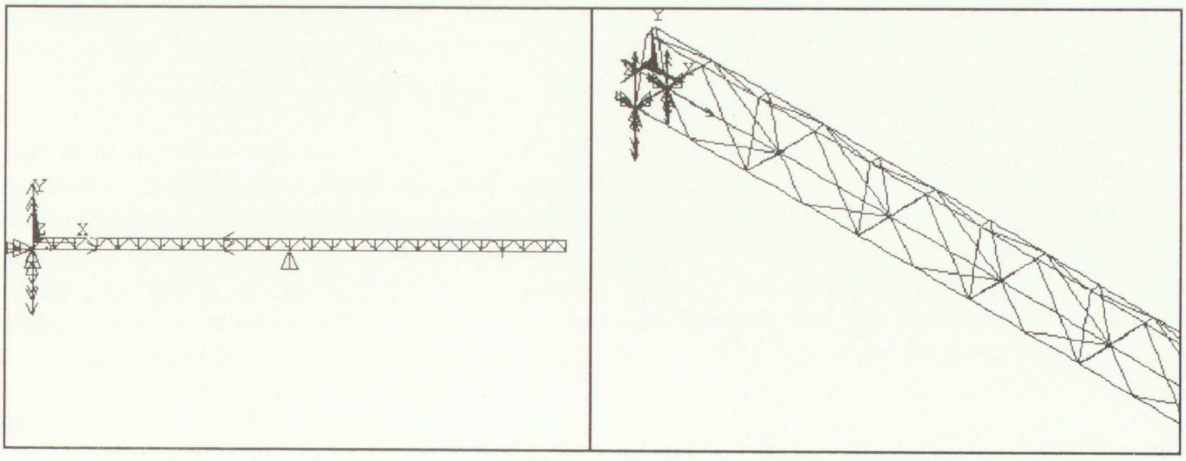


Figure 5 Analysis of the hoisting device lattice construction by means of the computer code based on the method of finite elements
Slika 5 Analiza dizalične rešetkaste konstrukcije softverom koji je baziran na metodi konačnih elemenata

This kind of optimization implied modeling and analysis of the basic model of the hoisting device lattice construction. In order to create finite element model of the hoisting device lattice construction, the computer code is written within the used computer program, which enabled automatic execution of iteration procedure for obtaining required results. This code enabled geometrical modeling, specification of material properties, application of loads, generation of finite element mesh, as well as solving and obtaining corresponding output data. In this case the output data are maximal stresses in corresponding cross sections.

Subsequently, the postprocessor analysis was carried out. In that analysis, first the independent variables were specified (dimensions of the construction cross section c, d and h and the flanges cross section areas A_i). Then the dependent variables (constraint function-stress criterion: maximal allowable stress in corresponding construction cross sections) and the objective function (minimal weight, i.e. minimal volume of analyzed model) were specified as well. Finally, the methods and tools of optimization execution were assigned.

For optimization purposes, the lattice construction was segmented into 17 sections. The cross section areas A_i, i=1..17 of those sections were different, and their length was 2m and 4m, except for the two last sections. Their length was, respectively, 2.8m and 5.2m.

For every variable, regardless if it was independent or not, the range of variation had to be specified. The range of variation specification independent variables was in the direct connection with the value of the maximal allowable stress, which in this case was 240MPa.

The range of variation specification of independent variables was more complicated. Too wide a range results in extend of optimization analysis, whereas too narrow a range may exclude optimal solutions. In this case, the obtained theoretical results were used for the range of variation of independent variables.

The created finite element model was adapted to the maximal opportunities that used computer program had to offer. It should be mentioned that calculation of optimal parameters required a lot of time. For example, just one optimization iteration could take up to 90 minutes, and 3 to 4

Ovakva optimizacija podrazumevala je najpre kreiranje i analiziranje osnovnog modela dizalične rešetkaste konstrukcije, što je zahtevalo pisanje programskog koda u okviru samog programa koji je baziran na metodi konačnih elemenata. Ovim kodom se, za osnovni model, izvršava geometrijsko modelovanje, zadavanje odgovarajućih osobina materijala, opterećenja, vrši se kreiranje mreže konačnih elemenata, reševanje i dobijanje odgovarajućih izlaznih veličina, a to su u ovom slučaju maksimalni naponi u odgovarajućim poprečnim presecima.

Zatim su se u postprocesorskoj analizi zadavale nezavisne ulazne promenljive (c, d i h geometrijske dimezije poprečnog preseka, kao i površine poprečnog preseka pojasa A_i), zavisne ulazne promenljive (funkcija ograničenja - kriterijum čvrstoće: maksimalni dozvoljeni napon u odgovarajućim poprečnim presecima), zatim funkcija cilja, (minimalna masa, tj minimalna zapremina konstrukcije), kao i metode i alati izvođenja optimizacije.

Za potrebe optimizacije celokupna konstrukcija je podeljena na sedamnaest segmenata čije su površine poprečnih preseka A_i, i=1..17 nezavisno promenljive, a čija dužina iznosi 2m i 4m, dok je dužina pretposlednjeg segmenta 2.8m a poslednjeg segmenta 5.2m.

Za svaku promeljivu, bez obzira da li je zavisna ili nezavisna, morao se zadati opseg u okviru koga se ona varirala. Definisanje opsega zavisno promenljivih je u direktnoj vezi sa veličinom maksimalnog dozvoljenog napona koji u ovom slučaju iznosi 240 MPa.

Definisanje opsega nezavisno promenljivih je bilo znatno komplikovanije, jer uzimanje prevelikog opsega u ogromnoj meri komplikuje proračun, dok nasuprot tome mali opseg može "ispustiti" optimalna rešenja. U ovom primeru za definisanje opsega nezavisno promenljivih su iskorišćeni podaci dobijeni predstavljenom teorijskom analizom.

Kreiran model konačnih elementa prilagođen je maksimalnim mogućnostima koje poseduje ovde korišćeni kompjuterski program. Značajno je pomenuti i da je izračunavanje optimalnih parametara zahtevalo dosta vremena. Tako je, na primer, za jednu iteraciju bilo potrebno oko 90

optimization iterations were necessary for only one load position.

The optimization was executed for different load positions along the construction and in the accordance with the obtained analytical expressions. This enabled comparison of the results.

minuta, a 3 do 4 iteracije su bile optrebne samo za jedan slučaj opterećenja.

Optimizacija je izvršena za različite položaje tereta, a u skladu sa dobijenim analitičkim izrazima, da bi se omogućilo poređenje rezultata.

9 COMPARISON OF THE RESULTS

The optimization results obtained by the computer program based on finite element analysis are represented on following graphs (figure 6), together with the results obtained by application of analytical expressions.

9 POREĐENJE REZULTATA

Rezultati dobijeni optimizacijom pomoću programa koji je baziran na metodi konačnih elemenata prikazani su na sledećim graficima (slika 6), zajedno sa rezultatima dobijenim primenom analitičkih izraza.

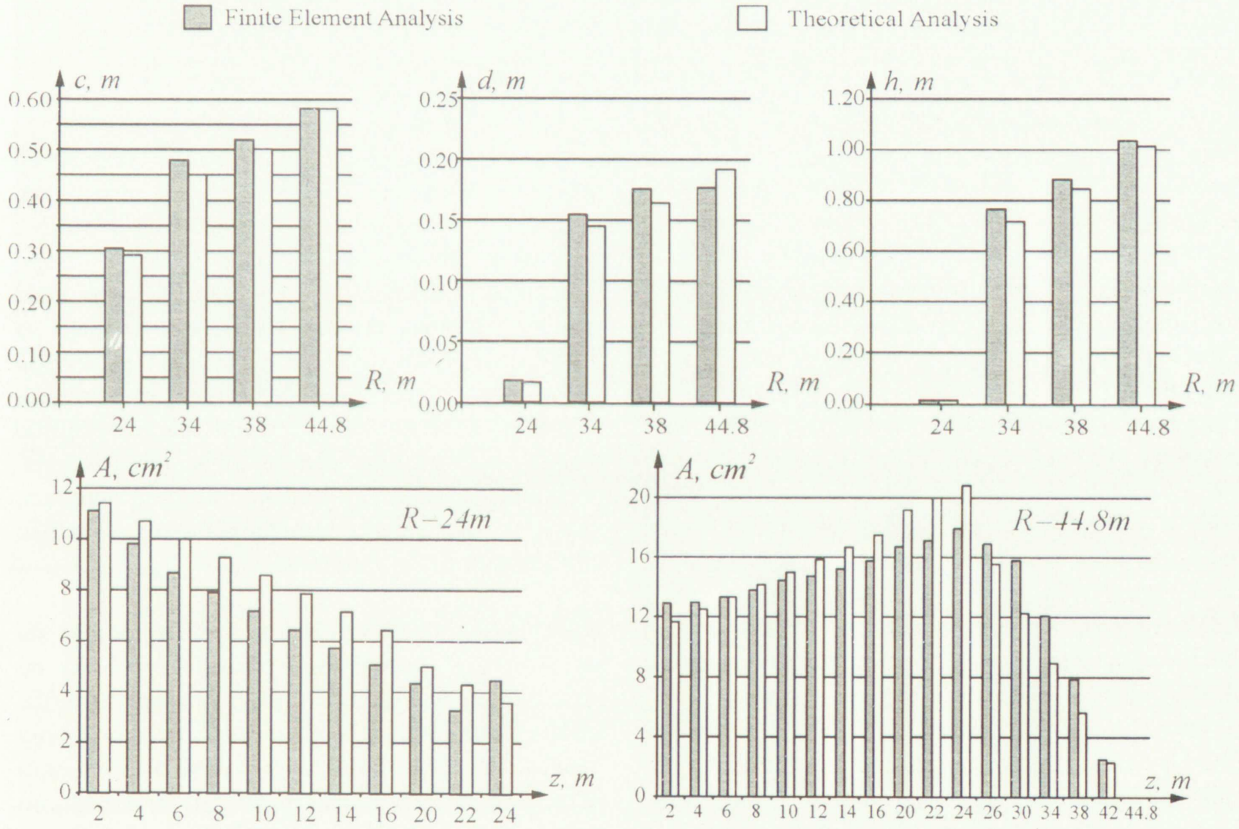


Figure 6 Comparison of optimal dimensions of construction cross section (c, d, h) and comparison of the optimal values of flanges cross section areas for different values of range (R)
Slika 6 Prikaz poređenja optimalnih vrednosti geometrijskih rastojanja poprečnog preseka (c, d, h) i optimalnih vrednosti površine poprečnog preseka pojasa za različite vrednosti dohvata (R)

Based on the comparison of the optimal dimensions of construction cross section obtained in these two manners, it can be concluded that the error does not exceed 10%, which is acceptable from an engineering point of view. However, grater error occurs in case of flanges cross section areas comparison. This error is caused by

Na osnovu poređenja geometrijskih karakteristika možemo zaključiti da je greška između rezultata dobijenih na ova dva načina, a koja se odnosi na optimalne vrednosti dimenzija poprečnog preseka, ispod 10%, što je zadovoljavajuće sa inženjerske tačke gledišta. Međutim, veća greška se dobija kada se porede površine poprečnog

the size of the segments on which the hoisting device construction was segmented, for the purposes of finite element analysis. Still, it can be observed that in places where the segments are smaller and in which proximity there are no characteristic cross sections (cross section with constrains or applied load), the error is lesser. This draws to conclusion that the increase of segments number and reduction of their size, especially in the proximity of characteristic cross sections, would result in significant decrease of error. However, the results obtained by the computer program based on finite element analysis are more then useful. They can be considered as starting results for further optimization of analyzed model and at the same time they emphasizes even more the significance of the obtained theoretical results.

10 CONCLUSION

The optimization of the cross section of the hoisting device lattice construction was performed in this paper. The analysis was carried out based on stress criterion. Equation (20)-(23) can be useful to an engineer-designer in the first stage of the designing procedure, when the problem of defining basic structure dimensions that would be close to optimal ones arise. Obtained results are compared with ones obtained by the computer program based on finite element analysis, whereby good agreement of the results was achieved, regarding the limitations of used computer program.

The established model included practical data that the overall dimensions of construction cross section (c, d, h) are constant along the construction. Based on this paper it can be concluded that it is necessary that flanges cross section area vary linearly with construction lengthwise coordinate (z), which is determined by expression (13), in order to achieve maximal material exploitation.

The directions of further research in this area are in the multicriteria analysis, which include additional constraint functions, like deformation criterion, dynamic stability criterion... From the exploration point, it would be convenient to determine optimal values of carrying capacity, regarding dimensions of construction cross section and dynamic exploitation parameters.

preseka pojasa. Ova greška je uzrokovana veličinom segmanata na koje je dizalična rešetkasta konstrukcija podeljena za potrebe analize konačnim elementima. Ipak, i kod ovakvih rezultata se može primetiti da tamo gde su segmenti manji i u čijoj blizini nema karakterističnih preseka (preseci u kojima su oslonci ili u kojima deluju opterećenja), tamo je i greška manja. Ovo upućuje na zaključak da bi povećanje ukupnog broja segmenata i njihovo "ugušćavanje" u okolini karakterističnih preseka, značajno povećalo tačnost i smanjilo grešku. Međutim, bez obzira na grešku, rezultati dobijeni programom koji je baziran na metodi konačnih elemenata su više nego korisni. Oni se mogu posmatrati kao polazni rezultati za dalju optimizaciju date konstrukcije, ali isto tako još više ističu značaj dobijenih teorijskih rezultata.

10 ZAKLJUČAK

U radu je ostvarena optimizacija poprečnog preseka dizalične rešetkaste konstrukcije. Sprovedena analiza je rađena na osnovu naponskog kriterijuma. Formule (20)-(23) mogu biti korisne inženjeru projektantu u prvoj fazi procesa projektovanja, kada je suočen sa problemima određivanja osnovnih dimenzija konstrukcije koje bi bile bliske optimalnim. Dobijeni rezultati su poređeni sa rezultatima dobijenim primenom metode konačnih elemenata, pri čemu je ostvareno dobro poklapanje rezultata, ako se u obzir uzmu ograničenja na koja se naišlo korišćenjem datog programa.

Postavljeni model prihvata praktičan podatak da se dimenzije poprečnog preseka (c, d, h) ne menjaju duž konstrukcije. Ovim radom se može zaključiti da je, u cilju maksimalnog iskorišćenja materijala, potrebno da je površina poprečnih preseka pojaseva u linearnoj vezi za podužnom koordinatom konstrukcije (z), a što je definisano izrazom (13).

Pravci daljih istraživanja iz ove oblasti su u višekriterijumskoj analizi uvođenjem dodatnih funkcija ograničenja, kao što su kriterijum deformacije, kriterijum dinamičke stabilnosti... Sa aspekta eksploatacije značajno bi bilo odrediti optimalnu vrednost dijagrama nosivosti od geometrijskih parametara dizalične rešetkaste konstrukcije i dinamičkih eksploatacionih parametara.

The proposed model of the hoisting device lattice construction can be applied on other lattice constructions as well, with minor modifications. Also, the model suggested in this paper can be applied, with minor modifications, for the determination of optimal design parameters of conveyor belts and other conveyor system for various applications.

Predloženi model dizalične rešetkaste konstrukcije se može uz manje modifikacije primeniti i na druge rešetkaste konstrukcije dizalica koje su namenjene za pretovar različitih tipova tereta. Takođe, prethodni model se može primeniti i za određivanje optimalnih dimenzija transportnih staza različite namene.

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