



MATEMATICKÉ VYJADRENIE OSI DRÔTU V TROJBOKOM PRAMENI OCEĽOVÉHO LANA

MATHEMATICAL EXPRESSION OF THE WIRE AXIS IN TRIHEDRAL STRAND OF STEEL ROPE

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Abstrakt: Príspevok sa zaoberá otázkou matematického vyjadrenia konštrukčných prvkov v trojbokom pramene. Popisuje odvodenie parametrických rovnic kriviek osí drôtov v pramene typu (3+9+15). Najprv je v každej vrstve vyjadrený jeden drôt, potom sú tieto rovnice zovšeobecnené pre ľubovoľný drôt v pramene.

Kľúčové slová: matematické vyjadrenie, trojboký prameň, matematické vyjadrenie

Abstract: This contribution deals with the question of mathematical expressing the construction elements in a trihedral strand. It describes the derivation of parametric equations of the curves of the wire axes in a strand of type (3+9+15). First, one wire is expressed in each layer, then the equations are generalized for an arbitrary wire in the strand.

Key words: mathematical expressing, trihedral strand, mathematical expressing

1 ÚVOD

Geometria konštrukcie oceľového lana vo veľkej miere ovplyvňuje jeho mechanické vlastnosti. Preto je potrebné venovať problematicke usporiadania a veľkosti konštrukčných prvkov v lane pozornosť. Pri riešení tejto problematiky je výhodné poznať ich matematické vyjadrenie. Budeme riešiť otázku matematického vyjadrenia drôtov

1 INTRODUCTION

The geometry of the construction of a steel rope influences its mechanical properties in a major way. Therefore, it is important to pay attention to the problems of the arrangement and the size of the construction elements in a rope. When solving these problems, it is profitable to know the mathematical expressions of these elements. We shall be solving the

v jednoducho vinutom trojbokom prameni.

2 GEOMETRICKÁ KONŠTRUKCIA PRAMEŇA

Uvažujme prameň kruhového lana 6(3+9+15)+v (STN 02 4361). Tento prameň je vytvorený z 3 drôtov jadra, 9 drôtov prvej vrstvy a 15 drôtov druhej vrstvy (**Obr.1**). Pre priemery drôtov , vo vrstvách platí: . Medzi drôtmami prvej vrstvy je medzera, medzi drôtmami druhej vrstvy medzera, medzi drôtmami susedných vrstiev neuvažujeme žiadnu medzera. Uhol vinutia drôtov v obidvoch vrstvách nech je .

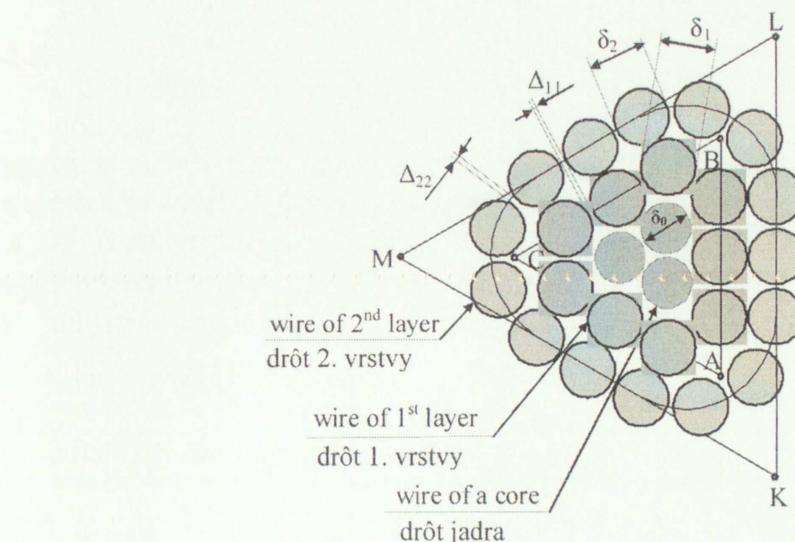
V reze prameňa body osí drôtov 1. vrstvy ležia na rovnostrannom trojuholníku ABC. Body osí drôtov 2. vrstvy ležia sčasti na stranach rovnostranného trojuholníka KLM a sčasti na kruhových oblúkoch vpísaných do tohto trojuholníka. Z popísaného geometrického usporiadania konštrukčných prvkov prameňa budeme vychádzať pri odvodzovaní matematického vyjadrenia osí jednotlivých drôtov.

question of the mathematical expression of wires in a simply wound trihedral strand.

2 GEOMETRIC CONSTRUCTION OF STRAND

Let us consider a strand of the circular rope 6(3+9+15)+v (STN 02 4361). The strand consists of 3 core wires, 9 first layer wires, and 15 second layer wires (**Fig.1**). For the diameters δ_1 , δ_2 of the wires in the layers it holds: $\delta_1 = \delta_2$. Between the wires of the first layer there is the gap Δ_{11} and between the wires of the second layer is the gap Δ_{22} . We assume that there is no gap between the wires of the neighboring layers. Let α be the winding angle of the wires in both layers.

In the cross-section of the strand, the points of the wire axes of the 1st layer lie on the equilateral triangle ABC. The points of the wire axes of the 2nd layer lie partially on the sides of the equilateral triangle KLM and partially on circular arcs inscribed into this triangle. The derivation of the mathematical expression of the axes of the particular wires shall be based on the above described geometric arrangement of the construction elements of the strand.



*Obr.1 Rez prameňom
Fig. 1 Cross-section of a strand*

3 MATEMATICKÉ VYJADRENIE OSI DRÔTU

Povrch drôtu v prameni tvorí cyklická skrutková plocha, ktorá vzniká transláciou kružnice $k(S;r)$ po osi drôtu. Kružnica pritom leží v normálovej rovine krivky osi a jej polomer $r = \frac{\delta}{2}$. Vyjadríme krivky osí drôtov prvej a druhej vrstvy pomocou parametrických rovnic.

3.1 Krivka osi drôtu 1. vrstvy

Pravotočivá karteziánska súradnicová sústava $(O; x, y, z)$ nech je umiestnená tak, že os z je totožná s osou prameňa o , a os x je kolmá na stranu AB trojuholníka ABC (fig. 2). S nech je bod osi drôtu, ktorej krivku vyjadríme. Predpokladajme, že drôt sa navija v smere pravom.

Časť SG krivky je úsečka. Jej parametrické rovnice pre $t \in \langle 0;1 \rangle$ sú:

3 MATHEMATICAL EXPRESSION OF THE WIRE AXIS

The surface of a wire in the strand is formed by a circle $k(S;r)$, which translates along the axis of the wire. The circle lies in the plane vertical to the wire axis and its radius $r = \frac{\delta}{2}$. We express the curves of the wire axes in the first and the second layer using parametric equations.

3.1 Curve of the wire axis of 1st layer

Let the right-handed Cartesian coordinate system $(O; x, y, z)$ be placed so that the z -axis is identical with the axis o , of the strand and the x -axis is perpendicular to the side AB of the triangle ABC (fig. 2). Let S be some point lying on the wire axis, whose curve we express. Let us assume that the wire is wound in the direction to the right. The part SG of the curve is a line segment. Its parametric equations for $t \in \langle 0;1 \rangle$ are:

$$x_{\text{u1}}(t) = \frac{2(\delta_1 + \Delta_{11})}{\sqrt{3}} t \quad (1)$$

$$y_{\text{u1}}(t) = (\delta_1 + \Delta_{11})(2t - 1) \quad (2)$$

$$z_{\text{u1}}(t) = 2 \cot \alpha (\delta_1 + \Delta_{11}) t \quad (3)$$

Časť GH krivky je skrutkovica. Aby nastalo hladké spojenie úsečky a skrutkovice, musia mať tieto v bode G spoločnú dotyčnicu. Preto os skrutkovice o_h , rovnobežná s osou z , prechádza bodom $X\left(\frac{\delta_1 + \Delta_{11}}{\sqrt{3}}; \delta_1 + \Delta_{11}; 0\right)$ (Obr. 3). Parametrické rovnice oblúka GH majú tvar:

The part GH of the curve is the helix. To obtain a smooth connection between the line segment and the helix segment, these two elements have to have a common tangent line in the point G . It follows that the axis o_h of the helix, which is parallel to the z -axis, goes through the point $X\left(\frac{\delta_1 + \Delta_{11}}{\sqrt{3}}; \delta_1 + \Delta_{11}; 0\right)$ (Fig. 3). The parametric equations of the arc GH have the form:

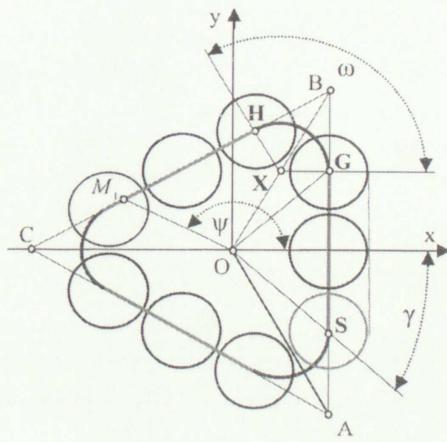
$$x_{\text{o1}}(\omega) = \frac{\delta_1 + \Delta_{11}}{\sqrt{3}} (\cos \omega + 1), \quad (4)$$

$$y_{\text{o1}}(\omega) = \frac{\delta_1 + \Delta_{11}}{\sqrt{3}} (\sin \omega + \sqrt{3}), \quad (5)$$

$$z_{\omega_1}(\omega) = \frac{\delta_1 + \Delta_{11}}{\sqrt{3}} \cot \alpha (\omega + 2\sqrt{3}), \quad (6)$$

kde $\omega \in \left(0; \frac{2}{3}\pi\right)$.

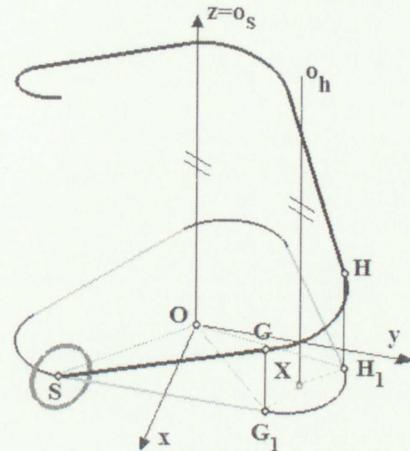
where $\omega \in \left(0; \frac{2}{3}\pi\right)$.



Obr. 2 Rez 1. vrstvou

Fig.2 Cross-section of the 1st layer wires

V nasledujúcich častiach krivky sa tieto dva segmenty opakujú pootočené o uhol $\kappa = k \frac{2\pi}{3}$ a posunuté o výšku $h = kh_{\Delta_1}$, kde:



Obr. 3 Drôt 1. vrstvy

Fig.3 Wire of the 1st layer

These two segments repeatedly appear in the subsequent sections of the curve where they are turned by the angle $\kappa = k \frac{2\pi}{3}$ and shifted by the height $h = kh_{\Delta_1}$, where:

$$h_{\Delta_1} = \cot \alpha \frac{(2\pi + 6\sqrt{3})(\delta_1 + \Delta_{11})}{3\sqrt{3}} \quad (7)$$

3.2 Krivka osi drôtu 2. vrstvy

Krivka osi drôtu 2. vrstvy je tak isto zložená zo segmentov priamky a skrutkovice. Predpokladajme, že $\Delta_{11} = \Delta_{22}$. Potom segment priamky vyjadrimo rovnicami:

3.2 Curve of the wire axis of 2nd layer

The curve of the axis of a wire of 2nd layer consists of line-segments and helical segments as well. Assume that $\Delta_{11} = \Delta_{22}$. The line-segment is then expressed by the equations:

$$x_{u_2}(t) = \frac{2(\delta_1 + \Delta_{11})}{\sqrt{3}} + \delta_1 \quad (8)$$

$$y_{u_2}(t) = (\delta_1 + \Delta_{11})(2t - 1) \quad (9)$$

$$z_{u_2}(t) = 2 \cot \alpha (\delta_1 + \Delta_{11}) t \quad (10)$$

pre $t \in \langle 0;1 \rangle$ a segment oblúka rovnicami:

for $t \in \langle 0;1 \rangle$ and the segments of the arc by the equations:

$$x_{\alpha_2}(\omega) = \frac{\delta_1 + \Delta_{11}}{\sqrt{3}} (\cos \omega + 1) + \delta_1 \cos \omega \quad (11)$$

$$y_{\alpha_2}(\omega) = \frac{\delta_1 + \Delta_{11}}{\sqrt{3}} (\sin \omega + \sqrt{3}) + \delta_1 \sin \omega \quad (12)$$

$$z_{\alpha_2}(\omega) = \cot \alpha \left[\frac{\delta_1 + \Delta_{11}}{\sqrt{3}} (\omega + 2\sqrt{3}) + \delta_1 \omega \right] \quad (13)$$

kde $\omega \in \left(0; \frac{2}{3}\pi\right)$.

where $\omega \in \left(0; \frac{2}{3}\pi\right)$.

Opakujúce sa časti krivky sú posunuté o výšku $h = kh_{\alpha_2}$, pričom:

The repeating sections of the curve are shifted by the height $h = kh_{\alpha_2}$, where:

$$h_{\alpha_2} = 2 \cot \alpha \left[(\delta_1 + \Delta_{11}) \left(1 + \frac{\pi}{3\sqrt{3}} \right) + \delta_1 \frac{\pi}{3} \right] \quad (14)$$

3.3 Krivka osi drôtu v prameni

V rovniciach (1) – (13) sa vyskytujú dva rôzne parametre. Vyjadríme krivku pomocou spoločného parametra ψ . Je to uhol, ktorý zviera pôdorys smerového vektora bodu M krivky s kladnou osou x (**Obr. 2**). Parameter t v rovniciach (1) – (3) a (8) – (10) nahradíme výrazom

3.3 Curve of the wire axis in the strand

There are two different parameters in the equations (1) – (13). We express the curve using single parameter ψ . This parameter is the angle between the vector \overrightarrow{OM} and the positive x -axis for any point M of the curve (**Fig. 2**). We replace the parameter t in the equations (1) – (3) and (8) – (10) by the expression:

$$t = \frac{2}{\sqrt{3}} \tan(\psi - \gamma) + 1 \quad (15)$$

a uhol ω je určený vzťahom:

and the angle ω is given by the expression:

$$\omega = \arccos \frac{(\sqrt{3}-1)\tan(\psi - \gamma) + \sqrt{2}[\sqrt{3}\tan(\psi - \gamma) - 1]}{\tan^2(\psi - \gamma) + 1} \quad (16)$$

kde

where

$$\gamma = \arctan \frac{\sqrt{3}}{2} \quad (17)$$

Potom rovnice vyjadrujúce krivku osi drôtu majú tvar:

The equations expressing the curve of the wire axis then have the following form:

$$x(\psi) = x_{ij}(\psi) \cos \kappa - y_{ij}(\psi) \sin \kappa \quad (18)$$

$$y(\psi) = x_{ij}(\psi) \sin \kappa + y_{ij}(\psi) \cos \kappa \quad (19)$$

$$z(\psi) = z_{ij}(\psi) + kh_{\Delta j} \quad (20)$$

kde $\psi \in \langle 0; 2\pi \rangle$ pre jednu výšku vinutia. Pritom pre $i=u$ je $\psi \in \langle \kappa; \kappa + 2\gamma \rangle$, pre $i=o$ $\psi \in \left\langle \kappa + 2\gamma; \kappa + \frac{2\pi}{3} \right\rangle$, $j=1$ pre drôt 1. vrstvy, $j=2$ pre drôt 2. vrstvy a v rovniciach (1) až (16) po úprave nahradíme uhol ψ uhlom $\psi_{\Delta} = \psi - \kappa$.

Osi ďalších drôtov vyjadrimo rovnicami (18), (19) a rovnicou:

where $\psi \in \langle 0; 2\pi \rangle$ for one height of the winding. Moreover, for $i=u$ we have $\psi \in \langle \kappa; \kappa + 2\gamma \rangle$, for $i=o$ we have $\psi \in \left\langle \kappa + 2\gamma; \kappa + \frac{2\pi}{3} \right\rangle$, $j=1$ for a wire of the 1st layer, $j=2$ for a wire of the 2nd layer and after modifying the equations (1) - (16), we replace the angle ψ by the angle $\psi_{\Delta} = \psi - \kappa$.

We express the axes of other wires using the equations (18), (19) and using the equation:

$$z(\psi) = z_{ij}(\psi) + kh_{\Delta i} + \frac{h_{\Delta}}{3} \quad (21)$$

pre drôty 1. vrstvy, rovnicou:

for the wires of the 1st layer, and using the equation:

$$z(\psi) = z_{ij}(\psi) + kh_{\Delta 2} + \frac{h_{\Delta 2}}{5} \quad (22)$$

pre drôty 2. vrstvy.

Uvedené rovnice sú použité na zobrazenie krviek osí drôtov na **Obr. 4**, **Obr. 6**, **Obr. 7** a plochy drôtu na **Obr. 5**.

Pomocou vhodného softvéru je možné s použitím niektorých z uvedených rovnic zostrojiť geometrický model prameňa.

for the wires of the 2nd layer.

The given equations are used to project the curves of the axes of wires on **Fig. 4**, **Fig. 6**, **Fig. 7** and the surface of a wire (**Fig. 5**).

Using suitable software and some of the above given equations, it is possible to construct a geometric model of the strand.

4. ZÁVER

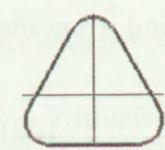
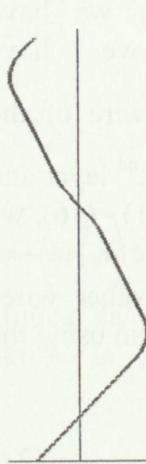
V príspevku sú matematicky vyjadrené krvky osí drôtov pre jednoducho vinutý trojboký prameň typu (3+9+15). Uvedené sú parametrické rovnice krvky osi drôtu 1. vrstvy a krvky osi drôtu 2. vrstvy, v ktorých sú použité dva rôzne parametre. Ďalej sú tieto rovnice upravené pre jeden parameter a zovšeobecnené pre ľubovoľný drôt vrstvy prameňa. Matematické vyjadrenie

4. CONCLUSION

In the contribution we give the mathematical expressions of the curves of the wire axes for a simply wound trihedral strand of type (3+9+15). We give parametric equations of the axis curve of a wire situated at the 1st layer and the axis curve of a wire situated at the 2nd layer, in which two different parameters are used. Subsequently, these equations are modified so that there is only one parameter and they are generalized for an arbitrary wire

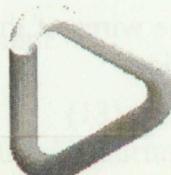
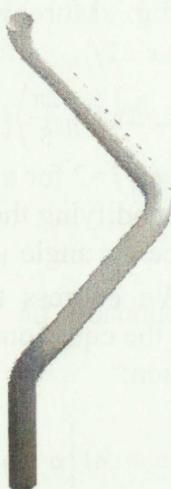
kriviek osí drôtov umožňuje zstrojíť geometrický model prameňa a pomocou neho riešiť niektoré otázky konštrukcie lana a jeho mechanických vlastností.

of a layer of the strand. The mathematical expression of the curves of the wire axes allows us to construct a geometric model of the strand and use this model to solve some questions concerning the construction of the rope and its mechanical properties.



Obr. 4 Os drôtu 1. vrstvy

Fig. 4 Axis of wire of the 1st layer



Obr. 5 Plocha drôtu Fig. 5 Surface of wire of the 1st layer

Fig. 5 Surface of wire of the 1st layer

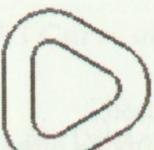
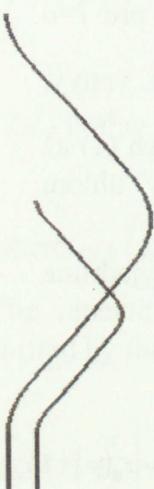


Fig. 6 Axes of wires of the 1st and the 2nd layer

Obr. 6 Osi drôtov v 1. 2 vrstvach



Fig. 7 Axes of wires of the 1st layer

Obr. 7 Osi drôtov v 1. vrstve

In this paper are used results from the project VEGA 1/4002/07 Surfaces in a geometrical modelling.

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