



## A METHODOLOGICAL AND PROCEDURAL FRAMEWORK FOR RANKING LINKS IN VULNERABLE ROAD NETWORKS

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**Abstract:** *This paper describes a methodologic and procedural framework for performing quantitative vulnerability analysis of a road network. Following advanced as well as consolidated approaches adopted by various well known transport scientists, road transport network vulnerability is interpreted as a variable that is directly related to link importance. Network simulation methods and software tools are proposed for estimating link flows and network cost differentials, under the quite sophisticated assumptions of Deterministic User Equilibrium instead of simply interpreting link costs to be constant with respect to link flows. Traffic counts are cheap data which are also suggested to be taken into consideration for both determining link vulnerability ranks and to surrogate possible lack of information regarding transport demand, which could be compensated, in total absence of data, by the application of proper gravitational models.*

**Key words:** *road networks, vulnerability analysis, link importance, network simulation, assignment process.*

## **1 INTRODUCTION**

In this paper, a methodological and procedural framework is described, which ranks the links of a network according to their importance for maintaining a proper connectivity between all origin-destination pairs. This method is helpful for prioritising ordinary and extraordinary maintenance investments to be planned along the links of a road transport network.

Indeed, road network vulnerability is a major determinant of risk in transport operations and, over the past twenty years, issues relating to the reliability and vulnerability of road networks have increasingly become a subject of considerable attention.

## **2 STATE OF PLAY**

The vulnerability of a certain element within a road network can be measured by quantifying the loss in the functionality of such an element, as a consequence of the occurrence of an accident with a given intensity. In other words, vulnerability corresponds to the sensitivity shown by either a single component or a complex entity with respect to its tendency to suffer damages. According to this definition, vulnerability can vary between 0 (no damage suffered) to 1 (total loss of functionality).

Some authors (Berdica, 2002) follow an approach, which considers vulnerability as a parameter strictly related to reliability. In particular, vulnerability in road transport systems is defined as “a susceptibility to incidents that can result in considerable reductions in road network serviceability; these incidents may then be more or less predictable, caused voluntarily or involuntarily, by men or nature”. Road vulnerability can be referred to a transport network as a whole or to distinct components, i.e. nodes and links. The serviceability of a link/route/network describes the possibility to use that link/route/network during a given time period. An incident is an event, which can result (directly or indirectly) in considerable reductions or interruptions in the serviceability of a road segment. Provided that risk depends on the probability of an incident to occur and the resulting consequences should the incident occur, reducing vulnerability can be regarded as reducing the risks involved in various incidents, either by reducing probabilities or reducing consequences.

Vulnerability in transport systems is often evaluated like in Jenelius et al. (2006) and Luathep et al. (2011), i.e. by adopting reliability theory, with probabilities that are regarded as inputs and consequences that are simulated in terms of variations of accessibility, connectivity and generalized trip costs. As reliability decreases, vulnerability will increase. Generally speaking network accessibility and connectivity are defined as the possibility to reach all destinations from one origin (or vice versa the possibility for one destination to be reached from all origins), possibly with a redundancy of routes. On the other hand, travel times are used for quantifying generalized trip costs and assumptions are typically made in calculations that travel times are independent of traffic load and travel demand is independent of travel times (only route choice is affected by variation in travel times).

It can be noted that the occurrence of an accident with partial or complete loss of functionality regarding some elements of a road transport network leads to an increase in travel times along the network. Moreover, as a consequence of the accident, the network may also be divided into several disconnected parts, i.e. the only path that connects some node pairs is interrupted, so that travel costs between such node pairs become infinite. According to this approach for estimating road network vulnerability, both transport supply and demand sub-systems (as well as their mutual interactions) are taken into account, and the simulations of transport networks in different conditions are required. On the contrary, topological

methods for evaluating vulnerability focus only on transport supply and do not consider transport demand, as they investigate the number of possibilities and the costs of routes linking all the nodes of the network each other.

Most measures of link importance, like those proposed by Jenelius et al. (2006), are derived from the difference between: the cost of travel from demand node  $i$  to demand node  $j$  when element  $e$  has failed and the cost of the initial undamaged network. In case cut links are present, i.e. when one or more links are closed and some o-d pairs will result to be disconnected, the measure must be limited only to non-cut links. On the other hand, measures based on unsatisfied demand will have to be calculated, being well defined with finite values for all links. The unsatisfied demand represents the number of trips that are unable to reach destinations from their origins as a consequence of the interruption of a given link.

### 3 METHODOLOGY

In this paper the vulnerability analysis of a road network is proposed to be conducted by adopting and integrating the approach proposed by Jenelius et al. (2006) based on measures of network reliability and link importance. In other words, a methodological and procedural framework is proposed aiming to answer the following question: which links are the most critical ones for the functioning of a mountainous road transport network? For this purpose, a set of relevant links are assumed to be successively and completely closed, which forces all travellers driving along such links to choose other and less advantageous routes.

In such a perspective, the vulnerability analysis of a road transport network corresponds to the design of a process for creating a rank of links according to their importance with respect to the preservation of network serviceability. Indeed, the most vulnerable components (for the functioning of the system as a whole) of a road transport network are represented by the most important links of the network, which are critical with respect to network accessibility as well as connectivity.

In brief, link importance depends on both topological characteristics and demand-related factors. In this study the following form for the index of link importance (LI) has been introduced to evaluate the importance of a generic link  $j$  for the functioning of the network as a whole:

$$LI(j) = f(ADT_j) + g(\Delta TC_j) \quad (1)$$

where  $f$  is a function that is directly proportional to  $ADT_j$ , i.e. the average daily traffic along link  $j$ , whose importance is under evaluation and  $g$  is a function that is directly proportional to  $\Delta TC_j$ , i.e. the increase in the network users' total cost due to the interruption of link  $j$  (calculated with respect to ordinary undamaged network configuration). Further details on the mathematical specification of LI index will not be provided in this paper. However, it deserves to be noticed that LI formula contains some calibration coefficients for the adaptation of the index itself to the particular network under study.

Regarding the practical calculation of LI index, a multi-step procedure has to be implemented in order to fix the input variables of such an index, which can be derived either directly or indirectly from the following main input data:

- traffic counts regarding the sub-set of links whose importance is to be determined within the network;
- the difference between network users' total cost in damaged and undamaged conditions, i.e. a suitable model for the simulation of the interaction between transport demand and supply.

Network simulation is needed in order to estimate the difference in total cost paid, i.e. time spent, to travel within the network by all vehicles. It has already been noted that for cut-links this index conventionally assumes an infinite value. Cut-links are particularly important in road networks serving mountainous areas, because this networks show a limited connectivity level and the number of cut-links, as a consequence, is relatively high.

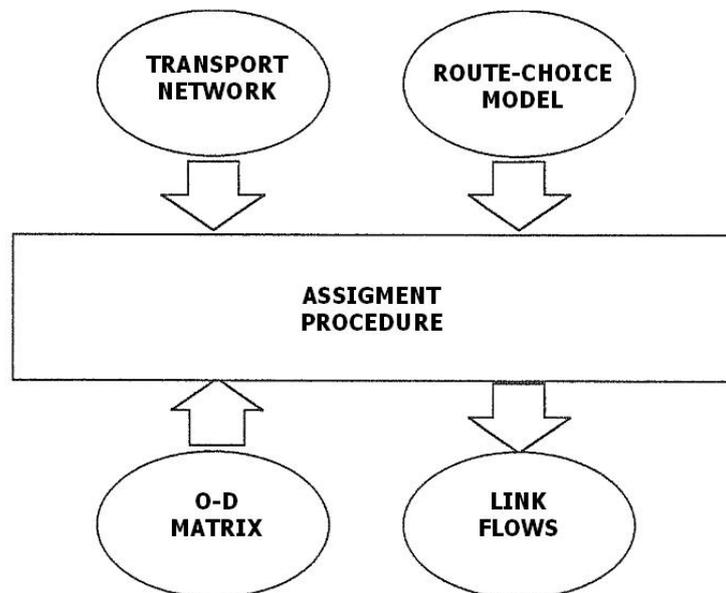
According to Cascetta (2008), the simulation of a transport network corresponds to the specification, calibration and validation of a model for assigning transport demand to the road network under study (Fig. 1). Indeed, in order to model a transport system the specification of three sub-models is required (Cascetta, 2008):

- model of transport supply sub-system (transport network in both damaged and undamaged conditions);
- model of transport demand sub-system (o-d matrix);
- demand-supply interaction model, i.e. a route-choice model, which computes link flows by assigning the o-d matrix to the network.

Firstly, the transport network, i.e. the model of transport supply sub-system, is constituted by a graph and array of link cost functions. The graph itself is composed by a set of nodes, which represent spatial and temporal positions occupied by traffic units within the system, and a set of links, which represent the existence of a relationship between two nodes. Cost functions are associated to network links indicating the average generalized trip cost paid by the users for travelling along the link.

Secondly, the transport demand model defines the number of trips made in a given time interval between each origin-destination pair within a network, leading to the so-called o-d matrix, which is characterised according to trip purpose, time window when trips are made, point of origin, point of destination and transport mode chosen for the trips.

Third, the assignment model is defined by the route choice rules adopted for the simulation of the interaction between transport demand and supply.



**Fig.1** - Demand-supply interaction model for the assignment demand to a transport network

The simulation of the initial undamaged road transport network provides as output an array of link flows as well as the average cost suffered by users along each link, whose most relevant component is travel time. Therefore the total cost of the network can be also

computed, being equal to the sum of generalized trip costs paid by all users. If one link is removed from the network, its importance can be estimated by measuring the increase in network total cost following this interaction. Then, a new simulation is needed, which assigns the o-d matrix to the damaged network where the same link is missing. In brief, such a procedure to estimate the increase in total cost for a damaged network includes the following steps:

- 1) the undamaged network is simulated and both traffic flows and the array of the generalized trip costs are estimated, so that the total cost on the undamaged network can be calculated, which is suffered by all network users when completing their trips;
- 2) link  $j$ , which belongs to the sub-set of links whose importance is to be calculated, is interrupted and, as a result a new model is obtained representing the damaged network missing the bi-directional link  $j$ ;
- 3) the  $j$ -th damaged network is simulated, leading to the calculation of the total costs characterising this network;
- 4) steps 2, 3 and 4 are repeated for all links included in the sub-set of links whose importance is to be calculated.

Following this procedure, the total cost of damaged networks obtained from the closure of cut-links will result in infinite values of total costs. These links will lead for their importance the list of links and another measure will need to be defined, taking into account the volume of unsatisfied demand. For the remaining non-cut links the difference between the total costs of the damaged and undamaged network can be properly calculated and link importance will be directly proportional to the increase in computed total cost.

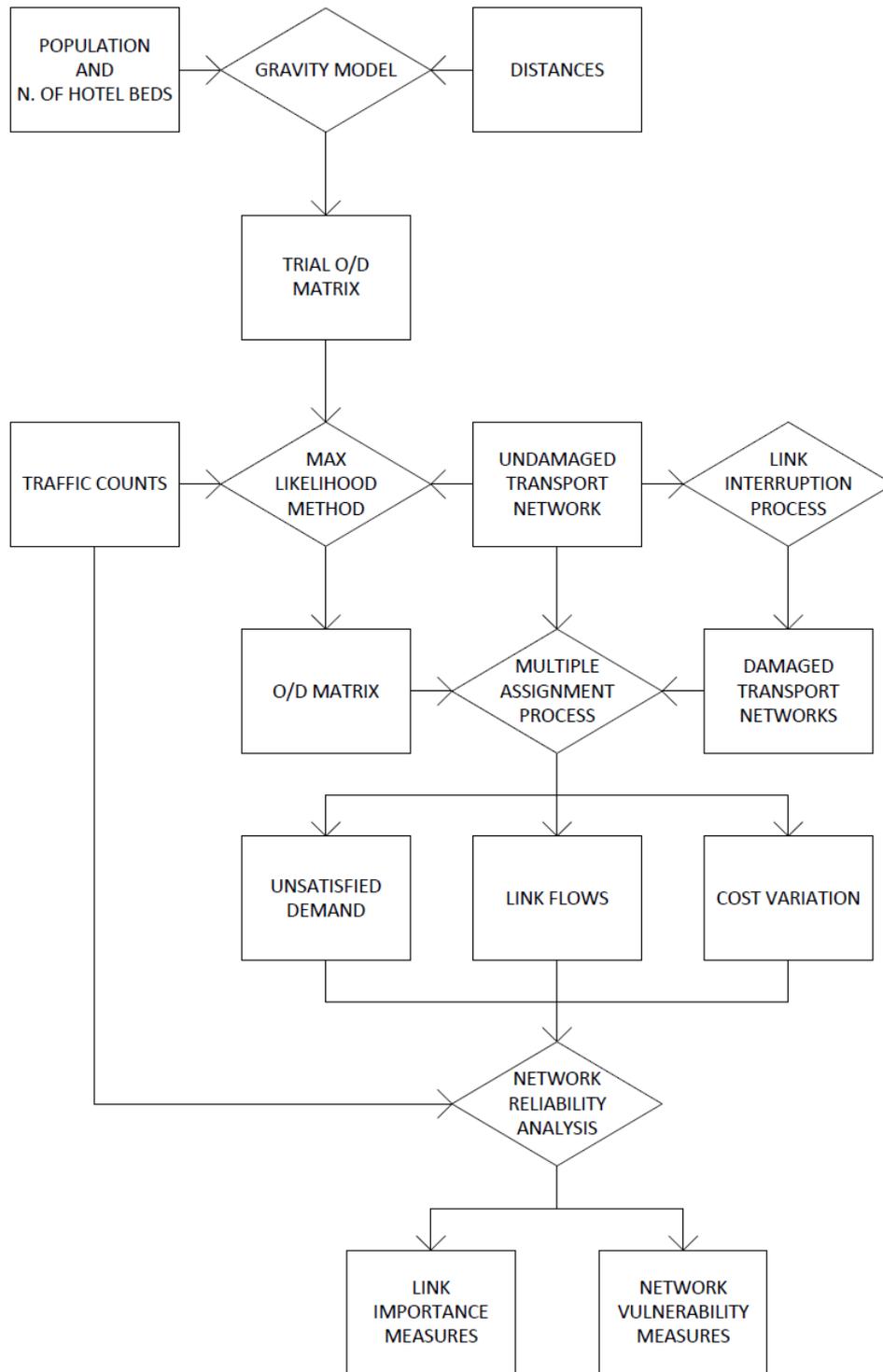
It can be observed that, in many practical applications, including the case under study, no reliable estimate of the o-d matrix is available. For this reason, traffic counts are needed, which often prove to be very helpful to correct the values of a trial matrix, by following a procedure based on the maximum likelihood method (Cascetta, 2008). The trial o-d matrix to be improved can be well represented by an old rough estimate or it can be even built up by defining a macroscopic demand model following the consolidated gravitational type (Kanafani, 1985). In the latter case, generation and attraction power of centroids nodes can be computed by considering socio-economic variables, whereas the impedance factor that is present between each o-d pair can be derived by cost-related measures proportional to distance, tariffs or other variables depending on the transport supply sub-system.

## **4 PROCEDURE**

In Fig. 2, a flow chart is represented to summarise the main steps of the procedure that is proposed to implement the described methodology.

### *4.1 Estimation of the O-D matrix*

First of all, an O-D matrix has to be estimated or derived starting from a rough preliminary version obtained by calibrating a gravitational demand model. The initial O-D matrix can be successively refined by taking into account traffic counts on a sub-set of road links in the network.



**Fig.2** - Flow chart describing the multi-step procedure adopted for performing the vulnerability analysis of the road network under study

The procedure that can be used for o-d matrix refinement starting from link traffic counts can be interpreted as a sort of inverse problem compared to the traditional assignment procedure. Indeed, on one hand, assignment models provide as outputs link traffic volumes by applying a route-choice rule to an o-d matrix and a transport network, which are needed as input data. On the other hand, the improvement of an o-d matrix estimate with traffic count data, incorporates as input a trial o-d matrix, the network model and traffic counts on a certain

set of links, while provides as output a new and more reliable estimation of the o-d matrix. The new matrix is found by solving an optimisation problem that minimise the objective function measuring the sum of the “distances” between the old and the new matrices and, simultaneously, the array of link traffic counts and link flows estimated by assigning the new matrix to the network according to a given route-choice model.

The abovementioned trial o-d matrix can be calculated by adopting a gravitational form that mimics the Newton gravitational law, where masses are replaced, for instance with a parameter taking into account population and number of hotel beds and distance is replaced by an impedance factor proportional to the line-distance between nodes, for each origin and destination.

#### *4.2 Multiple assignment process*

Regarding the simulation of the interaction between transport demand and supply, it has already been observed that demand-supply interaction models simulate traffic loads produced by the assignment of transport demand (O-D matrix) to the road network under study. A static approach could be adopted so that the methodology can be implemented through a relatively simple procedure referring to average constant conditions during a given time window (e.g. one average working day).

The core of traffic assignment models is represented by path-choice models, that can be distinguished between deterministic and stochastic models, on the basis of the hypotheses introduced with reference to users' perception of trip costs. On one hand, deterministic assignment models consider that no difference arises between users in the perception of trip costs, while stochastic models are based on a less simplified interpretation with a random distribution of trip costs around a mean value. The deterministic model could be adopted in order to facilitate practical implementation, which assumes that all users perceive generalized trip costs along the links in a constant and uniform way.

A further distinction can be made between network loading and user equilibrium assignment models. Network loading models assume that link costs remain constant and do not vary as a function of link volumes. This hypothesis can be accepted without losing precision mainly for uncongested road networks. On the other hand, equilibrium models take into account the variation of trip costs as a consequence of road network congestion. Indeed generalized trip costs do vary in accordance to the number of traffic units running along the links, depending on link capacity. The link-cost function that could be considered in the analysis is the BPR formula (Cascetta, 2008), where the time for running along a given link of the network is proportional to a power function of the ratio between link flow and link capacity, i.e. the maximum flow that can pass through a link section in a certain time window.

Regarding the multiple assignment process, it can start after a number of hypothetical damaged networks is generated by interrupting in each case one different bi-directional link. The multiple assignment process is completed in order to determine traffic volume configurations for all damaged networks and compare each configuration of the network to the case of no damaged links present.

In particular, unsatisfied demand and cost increases registered for each damaged network are simulated, so that an estimate can be obtained regarding the importance of the various links for network operations. The measures of importance of road links form the basis for network reliability and vulnerability analysis, as the road network can prove to be more or less vulnerable to disruptions in the functioning of different links.

At the end of the multi-step procedure performed, each link is labeled with an index, whose value ranges between 0 and 1, being proportional to the importance of the link for the network as a whole. Cut links, i.e. those links whose interruption leads to a non-zero value of

transport demand remaining unsatisfied, take the top of the importance list and follow a decreasing order of unsatisfied demand. Then, non-cut links, i.e. those links whose interruption do not generate unsatisfied demand, follow with a value of their label, which is proportional to the link traffic volume and to the total trip cost increase imposed to network users by the interruption of the link.

The results of the analysis can be reproduced in a graphical form, e.g. through Cube software or GIS modules, and directly read on a map.

## 5 CONCLUSIONS

In this paper a methodological and procedural framework is presented to perform vulnerability analysis of any road transport network whose operations may be jeopardised by natural hazards or external factors. Indeed a multi-step procedure has been implemented in order to decompose a complex problem into a series of sub-problems whose solutions have been targeted by adopting advanced as well as consolidated methods proposed in the scientific international literature.

The proposed methodology and procedure estimates the vulnerability of a road transport network by estimating the vulnerability of its links, which is assumed to be proportional to their importance for granting transport connections between each o-d pair, possibly with low consumption of time. Then, the variation of total network costs is calculated, due to the interruption of every link whose importance is to evaluate. Thus, the proposed approach is in line with the well known approach by Berdica (2002) and Jenelius et al. (2006), but network simulation is suggested to be performed with proper commercial tools for transport network analysis like Cube software. Cube software has been run by adopting Deterministic User Equilibrium (DUE) assumptions, i.e. taking into explicit consideration the dependence of link running times and costs by link traffic flows. In this way significant advantages can be obtained in terms of plausibility of network simulation results, while the increase in calculation complexity is almost fully absorbed by the software tool and not experienced by the user. Furthermore, the concept of link importance for the network as a whole is taken into account together with easy-to-measure data like average daily traffic along the link itself. Such an inclusion of both general and somewhat local determinants of link importance for road network vulnerability represents an innovation compared to the abovementioned approaches proposed in the scientific literature.

A further innovation that deserves to be remarked is that the multi-step procedure illustrated in this paper also indicates practitioners what to do in absence of reliable data regarding road transport demand. Indeed, the first phases of the procedure, methodological recommendations are implemented about (1) how to improve a rough estimate of the o-d matrix, by using data collected with traffic counts, as well as (2) how to build up a trial o-d matrix to improve in case no data about transport demand are available.

The methodology has been tested by transport engineers on real-scale networks with hundreds of elements, with satisfactory results represented by a rank of links, in decreasing order (from 1 to 0) of their vulnerability (and importance) scores. The results obtained can be easily read by practitioners and decision-makers either in an Excel table or incorporated in the user-friendly graphical interface of Cube software. Link vulnerability ranks can prove to be relevant, for instance, for the determination of a list of priorities in the allocation and/or orientation of economic resource for infrastructure maintenance and improvement.

### **References**

- [1] Berdica K.: An introduction to road vulnerability: what has been done, is done and should be done (2002) *Transport Policy*, vol.9, pp.117-127
- [2] Cascetta E.: *Transportation Systems Analysis: Models and Application*. (2008) Springer-Verlag New York Inc.
- [3] Jenelius E., Petersen T. and Mattsson L.G.: Importance and exposure in road network vulnerability analysis (2006) *Transportation Research Part A*, vol. 40, n.7, pp. 537-560
- [4] Kanafani A.K.: *Transportation Demand Analysis* (1983) McGraw-Hill
- [5] Luathep P., Sumalee A., Ho H.W. and Kurauchi F.: Large-scale road network vulnerability analysis: a sensitivity analysis based approach (2011) *Transportation*, vol. 38, pp. 799-817